Explanation of some physical concepts in the light of Uncertainty Relation:

* What is uncertainty relation?

If the commutator of two operators holds the relation

$$[A, B] = iC$$

then there exists an uncertainty relation between them which is expressed as,

$$\Delta A \, \Delta B \geq \frac{\langle C \rangle}{2}$$

Common form of this uncertainty relation is the position-momentum uncertainty principle.

$$\Delta x \, \Delta p_x \geq \hbar/2$$

where Δx is the uncertainty in determining the position and Δp_x is uncertainty in determining the momentum.

i.e. the precise values of both the position and momentum of a subatomic particle cannot be determined simultaneously.

Using uncertainty relation, some physical concepts may be explained.

(3) $n = 0$ is not allowed for a particle in a 1-D box.

For a PIB, $E = \frac{\langle p_x^2 \rangle}{2m} \approx \frac{\Delta p_x^2}{2m} = \frac{n^2 h^2}{8ma^2}$

If $n \to 0$, then E as well as $\Delta p_x \to 0$

From uncertainty principle,

$$\Delta p_x \, \Delta x \geq \hbar$$

Now if uncertainty in momentum is zero, then $\Delta x \to \infty$ which is not possible as the particle is bound to reside somewhere inside the box. Thus $n \neq 0$.

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① Validity of Bohr's atomic model

According to Bohr's postulate, e^-'s revolve round the nucleus in certain specific stationary circular orbits and angular momentum of the electron is

$$mvr = \frac{nh}{2\pi} \quad \text{where } n = 1, 2, 3, \cdots$$

$r \rightarrow$ radius of the Bohr orbit
$v \rightarrow$ velocity of the momentum.

Thus according to Bohr's model

$\Delta r \ll r$ as radius as well as position of the e^- is ~~too~~ precise.

$\Delta p \ll p$ as momentum is also ~~well~~ precisely measurable

$$\therefore \frac{\Delta r \, \Delta p}{rp} \ll 1$$

$$\Rightarrow \frac{\Delta p \, \Delta r}{pr} \ll 1 \quad \text{—①}$$

Heisenberg' uncertainty,

$$\Delta p \, \Delta r \geqslant \hbar$$

$$\Rightarrow \frac{\Delta p \, \Delta r}{pr} \geqslant \frac{\hbar}{pr}$$

$$\Rightarrow \frac{\Delta p \, \Delta r}{pr} \geqslant \frac{\hbar}{n\hbar}$$

$$\Rightarrow \frac{\Delta p \, \Delta r}{pr} \geqslant \frac{1}{n} \quad \text{—②}$$

Both ① & ② satisfies when $n \rightarrow \infty$

$\therefore$ Bohr's atomic model is valid only for large quantum numbers.

Also Bohr's correspondance principles is hereby validated that describes quantum-classical correspondence for large quantum no.s.

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Thus Bohr's orbit concept ~~is not~~ that suggests simultaneous accurate measurement of position & momentum is not acceptable. But in spite of that, 1st Bohr orbit is experimentally verified. This can also be explained by Uncertainty relation.

Total energy $E = K.E. + P.E.$

$$= \frac{p^2}{2m} - \frac{Ze^2}{r} \quad \text{—(1)}$$

$\Delta p \, \Delta r \geq \hbar$

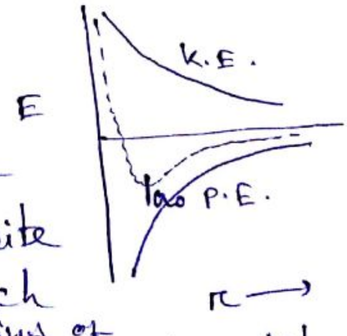
$\therefore \Delta p \geq \frac{\hbar}{\Delta r}$

Thus in terms of uncertainty relation Eq. (1) can be rewritten as

$$E = \frac{\hbar^2}{2m\Delta r^2} - \frac{Ze^2}{\Delta r} \quad \text{—(2)}$$

With increase in Δr, contribution of K.E. term to the total energy decreases and that of P.E. term increases.

So, there must be certain point/region where these two opposite effects compensates each other. That is the radius of Bohr's 1st orbit, a_0.



From (2), $$\frac{\partial E}{\partial(\Delta r)} = -\frac{\hbar^2}{m\Delta r^3} + \frac{Ze^2}{\Delta r^2} = 0$$

$$\Rightarrow \frac{1}{\Delta r^2}\left[Ze^2 - \frac{\hbar^2}{m\Delta r}\right]$$

$\frac{1}{\Delta r^2} \neq 0$ as Δr has a precise value

$$\therefore Ze^2 - \frac{\hbar^2}{m\Delta r} = 0$$

$$\therefore \Delta r = \frac{\hbar^2}{mZe^2} \quad \text{—(3)}$$

Bohr's radius $r_n = \frac{n^2\hbar^2}{mZe^2}$

For 1st Bohr orbit, $r = \frac{\hbar^2}{mZe^2}$ → identical with (3)

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(4) Electron cannot reside within ~~...~~ nucleus

Let electron is within the nucleus.

$\therefore$ Uncertainty in its position will be $\Delta x \simeq 10^{-14}$

$$\Delta p \, \Delta x = \hbar$$

$$\Rightarrow \Delta p \simeq \frac{10^{-34}}{10^{-14}} \simeq 10^{-20}$$

$$E^2 = p^2c^2 + (m_0 c^2)^2$$

$E \simeq 930$ MeV

But binding energy of electron is found to be 10 – 12 MeV

Hence electron cannot be in the nucleus.

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