To study the random error in observations

August 11, 2025

1 Definitions

Random error Variations in measured values due to unpredictable fluctuations in the experimental conditions or observer response. They cause scatter around the true value and average out over many measurements.

Relative error The ratio of the absolute error to the measured value, expressed as a percentage:

$$\text{Relative error} = \frac{\delta}{\overline{T}} \times 100\%.$$

Precision A measure of how closely repeated measurements agree with one another, quantified by the standard deviation or standard error of the mean. Higher precision means less scatter.

Error propagation The method of determining how uncertainties in measured quantities affect the uncertainty in a calculated result. For a function $g = f(x, y, \dots)$, the propagated random error is

$$\delta_g = \sqrt{\left(\frac{\partial f}{\partial x}\,\delta_x\right)^2 + \left(\frac{\partial f}{\partial y}\,\delta_y\right)^2 + \dots}.$$

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2 Theory

A simple pendulum of length ℓ and small amplitude has period

$$T = 2\pi\sqrt{\frac{\ell}{g}}.$$

When you measure T repeatedly, the collected values T_i follow approximately a Gaussian (normal) distribution:

$$f(T) = \frac{1}{\sigma\sqrt{2\pi}} \exp\Big[-\tfrac{(T-\mu)^2}{2\sigma^2}\Big],$$

where μ is the true mean and σ is the standard deviation.

Within one standard deviation (1σ) lies about 68.3% of values; within 2σ, 95.5%; and within 3σ, 99.7%. These "sigma" intervals allow you to judge how likely an outlier is due to random scatter.

By increasing the number of measurements N, the standard error of the mean

$$\delta_T = \frac{\sigma}{\sqrt{N}}$$

decreases as $1/\sqrt{N}$. Thus precision improves and random uncertainty shrinks.

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3 Experimental Design

- Collect two sets of period measurements:
  1. **Set 1:** $N = 20$ independent determinations of T.
  2. **Set 2:** $N = 60$ independent determinations of T.
- For each set:
  - Tabulate T_i.
  - Plot a histogram and overlay the Gaussian curve.
  - Compute $\overline{T}$, σ, δ_T, relative error.
  - Analyze how δ_T scales with N.

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4 Experimental Setup

- Rigid support stand with low-friction pivot
- Inextensible string of adjustable length ℓ
- Metallic bob (mass m)
- Stopwatch (0.01 s resolution)
- Meter-scale (1 mm resolution)
- Spirit level and protractor

Ensure the string swings in a single plane and the support is perfectly level.

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5 Procedure

Set 1: 20 Measurements

1. Measure ℓ from pivot to bob center.
2. Displace bob by $\theta_0 < 10^\circ$. Release without push.
3. Time $N_{\text{osc}} = 20$ oscillations; record total time t_i.
4. Compute each period $T_i = t_i / N_{\text{osc}}$.
5. Repeat for $i = 1$ to 20 to obtain $\{T_i\}$.

Sample Data (20 points):

Trial i	t_i (s)	$T_i = \frac{t_i}{20}$ (s)
1	35.74	1.787
2	35.80	1.790
...	...	...
20	35.72	1.786

Table 1: Twenty independent measurements of the pendulum period.

Set 2: 60 Measurements

Repeat steps 1–5 above for $i = 1$ to 60, yielding 60 period values $\{T_i\}$.

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6 Data Analysis

1. Compute mean:

$$\overline{T} = \frac{1}{N} \sum_{i=1}^{N} T_i.$$

2. Compute standard deviation:

$$\sigma = \sqrt{\frac{1}{N-1} \sum_{i=1}^{N} (T_i - \overline{T})^2}.$$

3. Compute standard error:

$$\delta_T = \frac{\sigma}{\sqrt{N}}.$$

4. Compute relative error:

$$\frac{\delta_T}{\overline{T}} \times 100\%.$$

5. Propagate to g:

$$g = \frac{4\pi^2 \ell}{\overline{T}^2}, \quad \delta_g = 2\,\frac{\delta_T}{\overline{T}}\,g.$$

6. Plot histogram of $\{T_i\}$ and overlay $\dfrac{1}{\sigma\sqrt{2\pi}} e^{-(T-\overline{T})^2/(2\sigma^2)}$.

7. Mark the 1σ, 2σ, 3σ intervals on the plot.

Comparison of Precision:

Data Set	N	$\overline{T}$ (s)	σ (s)	δ_T (s)	Rel. Err. (%)
Set 1	20	1.7879	0.0028	0.000 63	0.035
Set 2	60	1.7881	0.0028	0.000 36	0.020

Table 2: Precision comparison for the two data sets.

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7 Error Reduction & Precision Improvement

- Increase N to reduce δ_T and relative error.
- Automate timing (light gates) to eliminate reaction-time scatter.
- Maintain small θ_0 and level support to reduce systematic contributions.
- Use the same operator or apply a calibrated reaction-time offset.

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8 Discussion Questions

1. How do the 1σ, 2σ, 3σ intervals relate to data confidence?
2. Why does δ_T scale as $1/\sqrt{N}$?
3. Which improvements most effectively reduce human-reaction random error?
4. How would combining multiple observers' data affect precision?