

Boolean Algebra

Boolean algebra can be considered as an algebra that deals with binary variables and logic operations. Boolean algebraic variables are designated by letters such as A, B, x, and y. The basic operations performed are AND, OR, and complement.

The Boolean algebraic functions are mostly expressed with binary variables, logic operation symbols, parentheses, and equal sign. For a given value of variables, the Boolean function can be either 1 or 0. For instance, consider the Boolean function:

$$F = x + y'z$$

Boolean Algebra is used to analyze and simplify the digital (logic) circuits. It uses only the binary numbers i.e. 0 and 1. It is also called as **Binary Algebra** or **logical Algebra**. Boolean algebra was invented by **George Boole** in 1854.

Rule in Boolean Algebra

Following are the important rules used in Boolean algebra.

- Variable used can have only two values. Binary 1 for HIGH and Binary 0 for LOW.
- Complement of a variable is represented by an overbar (-). Thus, complement of variable B is represented as \bar{B} . Thus if B = 0 then $\bar{B} = 1$ and B = 1 then $\bar{B} = 0$.
- ORing of the variables is represented by a plus (+) sign between them. For example ORing of A, B, C is represented as A + B + C.
- Logical ANDing of the two or more variable is represented by writing a dot between them such as A.B.C. Sometime the dot may be omitted like ABC.

Boolean Laws

There are six types of Boolean Laws.

Commutative law

Any binary operation which satisfies the following expression is referred to as commutative operation.

$$(i) A.B = B.A \quad (ii) A + B = B + A$$

Commutative law states that changing the sequence of the variables does not have any effect on the output of a logic circuit.

Associative law

This law states that the order in which the logic operations are performed is irrelevant as their effect is the same.

$$(i) (A.B).C = A.(B.C) \quad (ii) (A + B) + C = A + (B + C)$$

Distributive law

Distributive law states the following condition.

$$A.(B + C) = A.B + A.C$$

AND law

These laws use the AND operation. Therefore they are called as **AND** laws.

$$(i) A \cdot 0 = 0$$

$$(ii) A \cdot 1 = A$$

$$(iii) A \cdot A = A$$

$$(iv) A \cdot \bar{A} = 0$$

OR law

These laws use the OR operation. Therefore they are called as **OR** laws.

$$(i) A + 0 = A$$

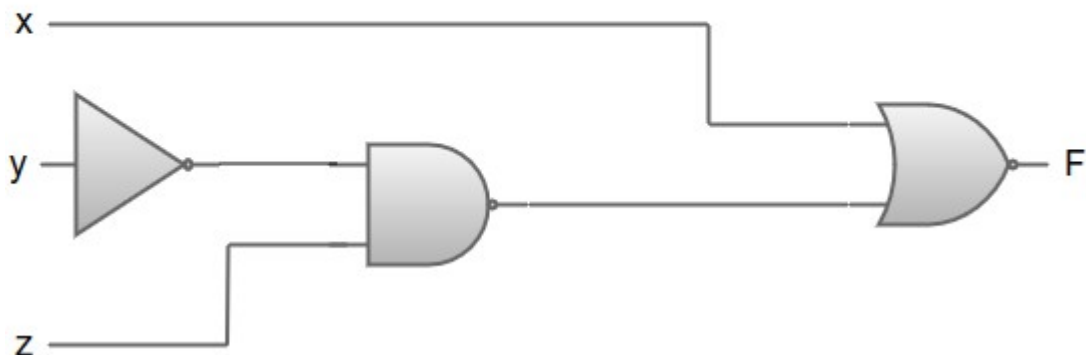
$$(ii) A + 1 = 1$$

$$(iii) A + A = A$$

$$(iv) A + \bar{A} = 1$$

The logic diagram for the Boolean function $F = x + y'z$ can be represented as:

$$F = x + y'z$$



- The Boolean function $F = x + y'z$ is transformed from an algebraic expression into a logic diagram composed of AND, OR, and inverter gates.
- Inverter at input 'y' generates its complement y' .
- There is an AND gate for the term $y'z$, and an OR gate is used to combine the two terms (x and $y'z$).
- The variables of the function are taken to be the inputs of the circuit, and the variable symbol of the function is taken as the output of the circuit.

A truth table can represent the relationship between a function and its binary variables. To represent a function in a truth table, we need a list of the 2^n combinations of n binary variables.

The truth table for the Boolean function $F = x + y'z$ can be represented as:

$$F = x + y'z$$

x	y	z	F
0	0	0	0
0	0	1	1
0	1	0	0
0	1	1	0
1	0	0	1
1	0	1	1
1	1	0	1
1	1	1	1