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## Boolean Algebra

A class of elements  $B$  together with two binary operations  $+$  and  $\cdot$  is a Boolean Algebra if and only if the following four postulates hold:

P<sub>1</sub>: Both operation  $+$  and  $\cdot$  are Commutative i.e for any two elements  $a$  and  $b$  of  $B$

$$a + b = b + a$$

$$a \cdot b = b \cdot a$$

Commutative Law

P<sub>2</sub>: Each of the operations  $+$  and  $\cdot$  are distributive over the other i.e for any three elements  $a, b$  and  $c$  in  $B$ ,

$$a + b \cdot c = (a + b) \cdot (a + c)$$

$$\text{and } a \cdot (b + c) = a \cdot b + a \cdot c$$

Distributive Law

P<sub>3</sub>: There exist in  $B$  distinct identity elements  $0$  &  $1$  ( $0 \neq 1$ ) for the operation  $+$  and  $\cdot$  respectively such that for any element  $a$  in  $B$

$$a + 0 = 0 + a = a$$

$$a \cdot 1 = 1 \cdot a = a$$

Identity Law

P<sub>4</sub>: For every element  $a$  in  $B$  there exist an element  $a'$  (Complement of  $a$ ) in  $B$  such that

$$a + a' = 1 \text{ and } a \cdot a' = 0$$

Complement Law

A Boolean algebra will be designated by a triple  $(B, +, \cdot)$