B.SC. COMPUTER SCIENCE SEM II

BOOLEAN ALGEBRA - NOTES 2

Q:) Explain Principle of Duality

Ans: By the dual of a statement in a Boolean Algebra, we mean a statement obtained from the first by interchanging the operation + and . and the identity elements 0 & 1 through out the statement or expression.

For example, the dual statement

$$xy + xy' + x'y + x'y' = 1$$ is given by

$$(x+y) \cdot (x+y') \cdot (x'+y) \cdot (x'+y') = 0$$

Principle of Duality: From the symmetry of the postulates with respect to the two operations + and . and two identity elements 0 & 1, it follows that if there is a valid statement or algebraic identity A in a Boolean algebra (B, +, ·) derivable from its basic postulates, then the dual of A is also derivable from these postulates and hence the dual is also a valid statement or algebraic identity. This is known as Principle of duality in Boolean algebra.

Contd..

Boolean Algebra: Some fundamental Theorems

T_1: (Idempotent Laws or Laws of Tautology)

For any element a in a ~~Bo~~ Boolean Algebra

(i) $a+a=a$ (ii) $a \cdot a = a$

Proof:-

$$
\begin{aligned}
R.H.S = a &= a+0 && \text{by } P_3 \\
&= a + a \cdot a' && \text{by } P_4 \\
&= (a+a)\cdot(a+a') && \text{by } P_2 \\
&= (a+a)\cdot 1 && \text{by } P_4 \\
&= a+a && \text{by } P_3 \\
&= L.H.S.
\end{aligned}
$$

So $a = a+a$ or $a+a=a$

(ii) From the principle of Duality we it can be obtained $a \cdot a = a$ from $a+a=a$

or we can also proof

$a \cdot a = a$

$$
\begin{aligned}
R.H.S = a &= a \cdot 1 && \text{by } P_3 \\
&= a\cdot(a+a') && \text{by } P_4 \\
&= a\cdot a + a\cdot a' && \text{by } P_2 \\
&= a\cdot a + 0 && \text{by } P_4 \\
&= a\cdot a && \text{by } P_3 \\
&= L.H.S
\end{aligned}
$$

So $a = a \cdot a$ or $a \cdot a = a$

Contd..

T_2: For any element a in a Boolean Algebra
(i) $a+1=1$ and (ii) $a.0=0$

Proof i) $a+1 = 1.(a+1)$ by P_3

$= (a+a').(a+1)$ by P_4

$= a + a'.1$ by P_2

$= a+a'$ by P_3

$= 1$ by P_4

(ii) $a.0=0$ is dual of i) $a+1=1$

T_3: Absorption Laws

(i) $a + a.b = a$ (ii) $a.(a+b) = a$

~~Proof~~ Proof (i) $a + a.b = a.1 + a.b$ by P_3

$= a.(1+b)$ by P_2

$= a.1$ by T_2(i)

$= a$ by P_3

ii) $a.(a+b) = a$ follows from (i) $a+a.b=a$ by principle of Duality

Home Task: Assignment 2

In a Boolean Algebra $(B,+,.)$ both the operations $+$ and $.$ are associative i.e for any three elements a, b and c of B

(i) $a+(b+c) = (a+b)+c$ and $a.(b.c) = (a.b).c$

This is known as T_4: Associative Laws