

Exercise

2.1

$A \vee (A \& B)$				$A \& (A \vee B)$				A
t	t	t	t	t	t	t	t	t
f	f	f	f	f	f	f	f	f
t	t	t	f	t	t	t	f	t
f	f	f	f	f	f	f	f	f

অন্তর্ভুক্ততা পদ্ধতির সাহায্যে প্রমাণিত করা যাচ্ছে, উপরোক্ত সারণী-দ্বারা
লজিকাল ভাবে সমতুল্যতা প্রমাণিত।

2.2

$$(A \vee B) \& (C \vee D), (A \& C) \vee (B \& C) \vee (A \& D) \vee (B \& D)$$

$$(A \vee B) \& (C \vee D)$$

$$\Leftrightarrow [(A \vee B) \& C] \vee [(A \vee B) \& D] \text{ - By distribution.}$$

$$\Leftrightarrow [C \& (A \vee B)] \vee [D \& (A \vee B)] \text{ - By commutation.}$$

$$\Leftrightarrow [(C \& A) \vee (C \& B)] \vee [(D \& A) \vee (D \& B)] \text{ - By distribution.}$$

$$\Leftrightarrow (A \& C) \vee (B \& C) \vee (A \& D) \vee (B \& D) \text{ - By commutation.}$$

Dist প্রক. comm প্রক. দ্বারা প্রমাণিত করা হয়েছে।

2.3

2.4

- a) $(M \& C) \vee -C$
- b) $C \vee (C \& M)$
- c) $M \vee (-M \& -C)$
- d) $M \vee -C \vee (C \& M)$
- e) $-(M \& C) \& (-M \vee -C)$
- f) $(M \& C) \vee (M \& -C)$
- g) $[(M \& C) \vee (-M \& -C)] \& (M \& C)$
- h) $-(M \& C) \vee -(-M \& -C)$

- a) \rightarrow ~~প্রমাণ~~ $(M \& C) \vee -C$
- b) $[H \vee (M \& C)] \& -C$
- c) $[H \vee \{(M \& C) \vee (-M \& -C)\}] \& C$

Simpler

2.3

a) $(m \& c) \vee -c$

$\leftrightarrow -c \vee (m \& c)$ - By commutation.

$\leftrightarrow -c \vee (c \& m)$ - By commutation.

$\leftrightarrow -c \vee m$ - By Absorption.

b) ~~$c \vee (c \& m)$~~

~~$\leftrightarrow c \vee m$ - By Absorption.~~

b) $c \vee (c \& m)$

$\leftrightarrow (c \& m) \vee (c \& -m) \vee (c \& m)$ - ~~by expansion~~

$\leftrightarrow (c \& m) \vee (c \& -m)$ - reduce

$\leftrightarrow c \& (m \vee -m)$ - dist

$\leftrightarrow c$ - Tautology dropped

c) $m \vee (-m \& -c)$

$\leftrightarrow m \vee -c$ - By Absorption.

d) $m \vee -c \vee (c \& m)$

$\leftrightarrow m \vee -c \vee m$ - By Absorption

$\leftrightarrow m \vee (-c \vee m)$ - By Association.

$\leftrightarrow m \vee (m \vee -c)$ - By commutation.

$\leftrightarrow (m \vee m \vee -c)$ - By Association.

$\leftrightarrow m \vee -c$ - By Redundancy.

e) $-(m \& c) \vee (-m \vee -c)$

$\leftrightarrow (-m \vee -c) \vee (-m \vee -c)$ - By distribution.

$\leftrightarrow -m \vee -c$ - By Redundancy.

f) $(m \& c) \vee (m \& -c)$

$\leftrightarrow m$ - By expansion.

g) $[(m \& c) \vee (-m \& -c)] \& (m \vee c)$

$\leftrightarrow [\{(m \& c) \vee -m\} \& \{(m \& c) \vee -c\}] \& (m \vee c)$ - by distribution

$\leftrightarrow [\{-m \vee (m \& c)\} \& \{-c \vee (c \& m)\}] \& (m \vee c)$ - by commutation.

$\leftrightarrow [(-m \vee c) \& (-c \vee m)] \& (m \vee c)$ - By Absorption.

$\leftrightarrow (-m \vee c) \& [(-c \vee m) \& (m \vee c)]$ - By Association.

$\leftrightarrow (-m \vee c) \& [m \vee (c \& -c)]$ - By distribution.

$\leftrightarrow (-m \vee c) \& m$ - By contradictory disjunct dropped.

$\leftrightarrow m \& (-m \vee c)$ - By commutation.

$\leftrightarrow (m \& -m) \vee (m \& c)$ - By distribution.

$\leftrightarrow m \& c$ - By contradictory disjunct dropped.

$$\begin{aligned}
& \neg (M \& C) \vee \neg (\neg M \& \neg C) \\
\leftrightarrow & \neg (M \& C) \vee \neg \neg (M \vee C) \text{ - By De.M.} \\
\leftrightarrow & \neg (M \& C) \vee (M \vee C) \text{ - By D.D.} \\
\leftrightarrow & (\neg M \vee \neg C) \vee (M \vee C) \text{ - By distrib De.M.} \\
\leftrightarrow & \neg M \vee (\neg C \vee M \vee C) \text{ - By Assoc.} \\
\leftrightarrow & \neg M \vee (C \vee \neg C \vee M) \\
\leftrightarrow & M \vee (C \vee \neg C \vee M) \text{ - By com} \\
\leftrightarrow & M \vee [(C \vee \neg C) \vee M] \text{ - By Assoc.} \\
\leftrightarrow & M \vee (C \vee \neg C) \text{ - Tautology of the disjunct dropped.} \\
\leftrightarrow & M \text{ - By of the.} \\
\leftrightarrow & \neg C \vee C \text{ - By Tautology of the disjunct dropped.}
\end{aligned}$$

2.4

b) $[H \vee (M \& C)] \& \neg C$

$$\begin{aligned}
& \leftrightarrow [(H \vee M) \& (H \vee C)] \& \neg C \text{ - By Dist.} \\
& \leftrightarrow \neg C \& [H \vee (M \& C)] \text{ - By com.} \\
& \leftrightarrow (\neg C \& H) \vee [\neg C \& (M \& C)] \text{ - By Dist.} \\
& \leftrightarrow (\neg C \& H) \vee [\neg C \& (C \& M)] \text{ - By com.} \\
& \leftrightarrow (\neg C \& H) \vee [(\neg C \& C) \& M] \text{ - By Assoc.} \\
& \leftrightarrow (\neg C \& H) \vee (\neg C \& C) \text{ - By contradictory conjunct dropped.} \\
& \leftrightarrow \neg C \& H \text{ - By contradictory disjunct dropped.}
\end{aligned}$$

c) $[H \vee \{(M \& C) \vee (\neg M \& \neg C)\}] \& C$

$$\begin{aligned}
& \leftrightarrow C \& [H \vee \{(M \& C) \vee (\neg M \& \neg C)\}] \text{ - By com.} \\
& \leftrightarrow (C \& H) \vee [C \& \{(M \& C) \vee (\neg M \& \neg C)\}] \text{ - By Dist.} \\
& \leftrightarrow (C \& H) \vee [(C \& (M \& C)) \vee (C \& (\neg M \& \neg C))] \text{ - By Dist.} \\
& \leftrightarrow (C \& H) \vee [(M \& C) \& C] \vee [(\neg M \& \neg C) \& C] \text{ - By com.} \\
& \leftrightarrow (C \& H) \vee [M \& (C \& C)] \vee [\neg M \& (\neg C \& C)] \text{ - By Assoc.} \\
& \leftrightarrow (C \& H) \vee [(M \& C) \vee (\neg C \& C)] \text{ - By Redundency and contradictory conjunct dropped.} \\
& \leftrightarrow (C \& H) \vee (M \& C) \text{ - By contradictory disjunct dropped.} \\
& \leftrightarrow (C \& H) \vee [(C \& M \& C) \vee (C \& \neg M \& \neg C)] \text{ - By Dist.} \\
& \leftrightarrow (C \& H) \vee [(C \& M) \vee (C \& \neg M \& \neg C)] \text{ - By Redundency.} \\
& \leftrightarrow (C \& H) \vee (C \& M) \text{ - By contradictory disjunct dropped.}
\end{aligned}$$