Chap – 10

Relation

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Binary Relation

What do you mean by cross product?

The cartesian or cross product of two sets we mean A and B $(A \times B)$ is the set of all ordered couples $\langle x, y \rangle$ such that $x \in A$ and $y \in B$

Example – $\{1,2\} \times \{3,4\} = \{\langle 1,3\rangle, \langle 1,4\rangle, \langle 2,3\rangle, \langle 2,4\rangle\}$

Binary relation is a set of ordered couples.
Every ordered couples have two elements. এদের order টা fixed.

$R = \{\langle 1,2\rangle, \langle 3,4\rangle, \langle 5,6\rangle\}$

$\langle 1,2\rangle \neq \langle 2,1\rangle$
$\langle 1,2\rangle = \langle 1,2\rangle$

The Domain of the Relation of R or $D(R) =$

$\{1, 3, 5\}$

The counter Domain of the relation of R or

$C(R) = \{2, 4, 6\}$

The Field of the relation of R ~~or~~ ~~Rot~~ or $F(R)$

$= \{1, 2, 3, 4, 5, 6,\}$

Domain + Counter Domain = Field.

Exercise P – 212

1) Let = $A_1 = \{1,2\}$
$A_2 = \{\Lambda\}$
$R = \{\langle 1,2\rangle, \langle 2,\Lambda\rangle\}$

a) R is not a subset of cartesian product $A_1 \times A_2$. $A_1 \times A_2 = \{\langle 1,\Lambda\rangle, \langle 2,\Lambda\rangle\}$ $\langle 1,2\rangle \in R$ but $\langle 1,2\rangle \notin A_1 \times A_2$. So R is not a subset of cartesian product $A_1 \times A_2$.

b) yes. $D(R)$ is a subset of A_1

$D(R) = \{1,2\}$ because all the members of $D(R)$ is a member of A_1

c) $C(R)$ is not a subset of A_2.

$C(R) = \{2, \wedge\}$ because $\{2\} \in C(R)$ but $\{2\} \notin A_2$.

d) yes. $F(R)$ is a subset of $A_1 \cup A_2$

$A_1 \cup A_2 = \{1, 2, \wedge\}$ because all the members of $F(R)$ are the members of $A_1 \cup A_2$.

$F(R) = \{1, 2, 2, \wedge\}$

2) The domain of the relation of being a father is the set of all fathers (all male persons having atleast one child).

The counter Domain of the relation of being a father is the set of all persons. since every one has a father.

The Field of the relation of being a father is the set of all persons.

3) The Domain of the relation of being a grand-father is the set of all grandfathers (all ~~the~~ ~~per~~ ~~male~~ male persons having atleast one grand child)

The counter domain of the relation of being a grandfather is the set of all persons, since everyone has a grand father.

The Field of the relation of being a grand father is the set of all grandfathers and grand-sons persons (all male persons having atleast one grand child and every one has a ~~grand~~ grand father).

4) yes The Domain of being a grand father is a proper subset of the domain of the relation of being a father. because all grand fathers are fathers ~~on~~ but not all fathers are ~~not~~ grand father.

[The counter domain of the relation of being a brother is the set of all ~~m~~ persons who have atleast one brother.

The Field of the relation of being a brother is the set of all male persons who have atleast one brother or sister as well as all persons who have atleast one brother].

<u>Properties of Binary relation</u> <u>P-213</u>

① <u>Reflexive Relation</u> – যে relation এ নিজের সাথে নিজের থাকে তবে হবে reflexive relation.

A relation R is reflexive in a set A $\Leftrightarrow (x)(x \in A \rightarrow xRx)$

$$\left.\begin{array}{l} 2 \leq 2 \\ 2 \leq 3 \end{array}\right\} \text{reflexive.}$$

প্রতিটি member যদি less than equal to এ নিজের সাথে নিজের হবে তখন reflexive হবে।

$\subseteq$ (subset) এর relation এ হবে reflexive.

In the set of all sets হবে reflexive.

$A = \{1, 2, 3\}$

$R = \{\langle 1,1\rangle, \langle 2,2\rangle, \langle 3,3\rangle\}$ (Reflexive relation)

$A = \{1, 2, 3\}$

$R_1 = \{\langle 1,2\rangle, \langle 2,1\rangle, \langle 1,1\rangle, \langle 3,1\rangle, \langle 1,3\rangle, \langle 2,2\rangle\}$

reflexive नहीं।

The relation R_1 is not reflexive because $3 \in A$ but $\langle 3,3\rangle \notin R_1$

② **Not reflexive**

$$-(x)[x \in A \rightarrow xRx]$$

$$\Leftrightarrow (\exists x)[x \in A \rightarrow -xRx]$$

यदि रिक्त सेट हो — $A = \emptyset$

$R = \{\langle 1,2\rangle\}$

Find whether R is reflexive in A or not?

Any relation R is reflexive in an empty set because the detination is satishfied vacuously.

Reflexive तो Not reflexive हो न सकते contradictory relation.

③ **Irreflexive relation** –

A relation R is irreflexive in set A $\Leftrightarrow$

$(x)[x \in A \rightarrow -xRx]$

$A = \{2, 3\}$

* $R_1 = \{\langle 1,2\rangle, \langle 1,1\rangle\}$ irreflexive.

$R_2 = \{\langle 1,2\rangle, \langle 2,2\rangle\}$ not reflexive.

The relation R_2 is neither reflexive nor irreflexive in set A.

The Relation R_1 is irreflexive and not reflexive in the set A.

④ **Not irreflexive** – $-(x)[x \in A \rightarrow -xRx]$

$$\Leftrightarrow (\exists x)[x \in A \wedge xRx]$$

$B = \{1\}$

* R_1 = not reflexive. R_2 = not reflexive and irreflexive

$e = \Lambda$ হলে R_1 R_2 দুটাই আবার reflexive এবং irreflexive হবে।

⑤ Symmetric relation –

A relation R is symmetric in A $\leftrightarrow (x)(y)[x \in A \,\&\, y \in A \,\&\, xRy \rightarrow yRx]$.

$A = \{1, 2\}$

$R_1 = \{\langle 1,2 \rangle, \langle 2,1 \rangle\}$ symmetric relation.

But $R_2 = \{\langle 1,2 \rangle, \langle 1,1 \rangle\}$ not symmetric because $\langle 1,2 \rangle \in R_2$ but $\langle 2,1 \rangle \notin R_2$. Again $R_3 = \{\langle 1,1 \rangle, \langle 2,2 \rangle\}$ is symmetric. (Definition X 3। তে I নং. 2 –টো)।

⑥ Not symmetric

$- (x)(y)[x \in A \,\&\, y \in A \,\&\, xRy \rightarrow yRx]$

$A = \{1, 2, 3, 4\}$

$R_1 = \{\langle 1,2 \rangle, \langle 2,1 \rangle, \langle 3,4 \rangle\}$ not symmetric

$R_2 = \{\langle 1,5 \rangle, \langle 2,6 \rangle\}$ symmetric (vacuously satisfied)

$R_3 = \{\langle 2,2 \rangle, \langle 1,1 \rangle\}$ symmetric.

$A = \{1\}$ হলে R_1, R_2 R_3 সবাই symmetric হবে,

$A = \{\Lambda\}$ হলেও $R_1 - R_3$ symmetric হবে,

All binary relations are symmetric in an empty set (Λ) and also in a set having ~~one~~ only one member.

$x \leq y = (x)(x \leq x)$ reflexive.

$x < y$ or $x \leq y$ symmetric হবে না,

⑦ Asymmetric relation – $(x)(y)[x \in A \,\&\, y \in A \,\&\, xRy \rightarrow -(yRx)]$

$A = \{1, 2, 3, 4\}$

$R_1 = \{\langle 2,3 \rangle, \langle 5,4 \rangle\}$ – Not symmetric and asymmetric

$R_2 = \{\langle 2,3\rangle, \langle 3,4\rangle, \langle 4,1\rangle\}$ - asymmetric + not symmetric.

[$A = \Lambda$ হলে R = অপ্রতিসম হবে ~~ও~~ reflexive নয়. ~~no~~ irreflexive হবে].

(8) Not asymmetric -

$$-(x)(y)[x \in A \,\&\, y \in A \,\&\, xRy \Rightarrow -(yRx)]$$
$$\leftrightarrow (\exists x)(\exists y)[x \in A \,\&\, y \in A \,\&\, xRy \,\&\, yRx]$$

$R_3 = \{\langle 5,6\rangle\}$ symmetric নয়. asymmetric.

$A\{1, 1, 2,$
$(1,2)\ (2,1)$
$(1,1)\ (1,1)$

(9) Antisymmetric

~~A~~ A ⇔ $(x)(y)[x \in A \,\&\, y \in A \,\&\, xRy \,\&\, yRx \rightarrow x = y]$

$R_1 = \{\langle 1,1\rangle, \langle 2,2\rangle\}$ Antisymmetric in A.

$R_2 = \{\langle 2,3\rangle, \langle 3,4\rangle, \langle 4,3\rangle\}$ not symmetric, not asymmetric but antisymmetric.

$R_3 = \{\langle 2,3\rangle, \langle 3,4\rangle, \langle 4,1\rangle\}$ antisymmetric

$R_4 = \{\langle 5,6\rangle\}$ Asymmetric (vacuously) vacously.

All asymmetric relation are ~~vacous~~ vacuously antisymmetric হবে,

(10) Not antisymmetric -

$$-(x)(y)[x \in A \,\&\, y \in A \,\&\, xRy \,\&\, yRx \rightarrow x = y]$$

The relation of being a father - Not symmetric, asymmetric, antisymetric.

The relation of being a cousin symmetric.

The relation of being a brother in the set of all male persons - Not antisymmetric, Not reflexive, Symmetric.

The relation of being a mother – asymmetric, antisymmetric.

xRy such that $x < y$ [the set of all possitive integers].

$\{ \langle 1,2\rangle, \langle 1,3\rangle, \langle 1,4\rangle ---- \}$

$\{ \langle 2,3\rangle, \langle 2,4\rangle, \langle 2,5\rangle ---- \}$ not symmetric, asymmetric & anti symmetric.

xRy such that $x < y+1$

$\{ \langle 1,1\rangle, \langle 1,2\rangle, \langle 1,3\rangle, \langle 1,4\rangle, \langle 2,2\rangle, \langle 2,3\rangle, \langle 2,4\rangle \}$ Reflexive হয়, not symmetric, না anti symmetric হয়;

$x > y+1$

$= \{ \langle 2,1\rangle, \langle 3,2\rangle, \langle 3,1\rangle, \langle 4,2\rangle, \langle 4,1\rangle, \langle 5,3\rangle \}$

not symmetric, asymmetric, and antisymmetric.

~~not~~ $x \geq y = \{ \langle 2,2\rangle, \langle 3,3\rangle, \langle 3,2\rangle. \langle 4,2\rangle \}$

Reflexive, antisymmetric, not symmetric

$x > y+1 = \{ \langle 3,1\rangle, \langle 4,2\rangle, \langle 4,1\rangle, \langle 5,3\rangle ---- \}$

not symmetric, not reflexive, asymmetric antisymmetric.

(11) Transitive relation (vacuously হয়েছে).

$A \Leftrightarrow (x)(y)(z)[x \in A \,\&\, y \in A \,\&\, z \in A \,\&\, xRy \,\&\, yRz \rightarrow xRz]$

$A = \{1, 2, 3, 4\}$

$R_1 = \{ \langle 1,2\rangle, \langle 2,3\rangle, \langle 1,3\rangle \}$ transitive relation.

$R_2 = \{ \langle 1,2\rangle, \langle 2,3\rangle, \langle 3,2\rangle, \langle 1,3\rangle, \langle 2,2\rangle, \langle 3,3\rangle \}$

transitive হয়,

(12) Not transative –

$-(x)(y)(z)[x \in A \ \& \ y \in A \ \& \ z \in A \ \& \ xRy \ \& \ yRz \rightarrow xRz]$

$A = \{1, 2, 3, 4\}$

$R = \{\langle 1,2\rangle, \langle 2,3\rangle, \langle 3,2\rangle, \langle 1,3\rangle\}$

not transitive.

(13) Intrasitive –

$A \Leftrightarrow (x)(y)(z)[x \in A \ \& \ y \in A \ \& \ z \in A \ \& \ xRy \ \& \ yRz \rightarrow -xRz]$

$A = \{1, 2, 3, 4\}$

$R_1 = \{\langle 1, 6\rangle, \langle 2, 8\rangle\}$ Transitive vacuously.

R6 Intransitive হবে, কারণ $x \in A$ but $y \notin A$.

$R_6 = \{\langle 1,2\rangle, \langle 2,3\rangle, \langle 2,4\rangle\}$ – intransitive

(14) R conected in A $\leftrightarrow$

$(x)(y)[x \in A \ \& \ y \in A \ \& \ x \neq y \rightarrow xRy \vee yRx]$

একটি relation connected হতে পারবে যদি set এর যেকোনো দুটি member এর মধ্যে relation থাকবে হয় x হতে y হবে বা y হতে x হবে,

$A = \{1, 2\}$

$x > y$ The relation of being greater than is connected the set of all possitive integers.

$x < y + 1$

$1 < 1 + 1$

$1 < (2 + 1)$ এই relation–ও connected

(15) Not connected -

$-(x)(y)[x \in A \, \& \, y \in A \, \& \, x \neq y \rightarrow xRy \vee yRx]$

strongly connected এর ক্ষেত্রে $x \neq y$ থাকবে না,
connected ও হবে ও reflexive ও হবে,

$(x)(y)[x \in A \, \& \, y \in A \rightarrow xRy \vee yRx]$

$A = \{1, 2, 3, 4$ —— $\}$ the set of all positive integers.

$R = \{\langle 1,1 \rangle, \langle 1,3 \rangle, \langle 2,2 \rangle, \langle 2,3 \rangle\}$ strongly connected

The relation of being a sister in the set of all persons — Not connected.

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1) a) The relation of being a grandfather in the set of all persons — Not reflexive, Irreflexive, Not symmetric, asymmetric, as antisymmetric, Not transitive, intransitive, neither connected nor strongly connected.

b) A = the set of all real numbers.
[All real numbers are rational number.]
Reflexive relation, not symmetric, not asymmetric,
[reflexive হলে asymmetric হবে না।]

~~x < y + +~~ $x < y$ connected হবে।

A = The set of all positive integers.
R = The relation of a less than.

$xRy \Leftrightarrow x < y$ connected

$xRy \Leftrightarrow \leq y$

$A = \{1, 2, 3, 4, 5\}$

$R = \{\langle 1,2 \rangle, \langle 3,4 \rangle\}$ not connected.

প্রতিটি member এর সাথে প্রতিটি member এর relation থাকলে তাকে connected বলা যাবে, $R = \{\langle 1,1 \rangle, \langle 2,2 \rangle, \langle 3,3 \rangle, \langle 4,4 \rangle, \langle 5,5 \rangle\}$ strongly connected.

$x \leq y$ connected হবে কিন্তু strongly connected হবে না।

$A = \{1, 2, 3, 4, 5\}$

$R = \{\langle 1,7\rangle, \langle 1,8\rangle\}$ not connected vacuously symmetric

$R1 = \{\langle 9,7\rangle, \langle 6,8\rangle\}$ "

connected হওয়ার vacuously satisfied হবে না।

$A = \{1\}$ হলে বা $A = \Lambda$ হলে যেকোন relation connected হবে না।

$A = \Lambda$ হলে strongly connected হবে।

$x < y + 1$

$= \{\langle 1,1\rangle, \langle 1,2\rangle, \langle 1,3\rangle, \langle 2,2\rangle, \langle 2,3\rangle, \langle 2,4\rangle ------ \}$

b) reflexive, not symmetric, not asymmetric, antisymmetric, Transitive, connected, strongly connected. (vacously)

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c) Not reflexive, irreflexive, not symmetric, asymmetric, anti-symmetric, ~~not~~ transitive, not connected, not strongly connected.

d) Not reflexive, irreflexive, not asymmetric, not symmetric, not antisymmetric, Not transitive, ~~not con~~ neither connected nor strongly connected.

e) Reflexive, symmetric, not asymmetric, antisymmetric, Transitive, not connected, not strongly connected. because $3 \in A$ & $5 \in A$ but neither 5 is related to 3 nor 3 is ~~not~~ related to 5.

f) Not reflexive because $2 \in A$ but $\langle 2, 2\rangle \notin R$, irreflexive, Not symmetric for $\langle 1,2\rangle \in R$ but $\langle 2,1\rangle \notin R$, ~~as~~ asymmetric, antisymmetric, Transitive, neither connected nor strongly connected.

g) Reflexive, symmetric, Not antisymmetric, Transitive, neither connected nor strongly connected, Not asymmetric

h) Not reflexive, irreflexive, not symmetric, asymmetric, antisymmetric, neither connected nor strongly connected, not transitive, intransitive.

i) Reflexive, Not symmetric, antisymmetric, Transitive, neither connected nor strongly connected, not asymmetric.

2) $\cancel{(x)(y)}\ (x)(y)[x \notin A \vee y \notin A \vee -(xRy)]$

3) $A = \{1, 2, \{1\}\}$

a) $R = \{\langle 1,1\rangle, \langle 2,2\rangle, \langle \{1\},\{1\}\rangle, \langle 1,2\rangle \langle 2,\{1\}\rangle, \langle 1,\{1\}\rangle\}$

b) $R = \{\langle 1,1\rangle, \langle 2,2\rangle, \langle \{1\},\{1\}\rangle, \langle 1,2\rangle, \langle 2,1\rangle, \langle 2,\{1\}\rangle, \langle \{1\},2\rangle\}$

c) $R = \{\langle 1,1\rangle, \langle 2,2\rangle, \langle \{1\},\{1\}\rangle, \langle 1,2\rangle, \langle 2,\{1\}\rangle\}$

$R = \{1,2\}$ — এটা vacuously transitive হয়েছে।

d) $R = \cancel{\{\langle 1,2\rangle\}}\ \{\langle 1,2\rangle, \langle 2,\{1\}\rangle\}$

4) $A = \{N, W\}$

$R = \cancel{\{\langle N,W\rangle, \langle W,N\rangle, \langle N,N\rangle}\ \{\langle N,1\rangle, \langle W,1\rangle\}$ vacuously

6) The relation of being a wife

5) $A = \{$Scott, the author of Waverly, Plato, Aristotle$\}$

$R = \{\langle S,$ the author of Waverly$\rangle, \langle$the author of Waverly$, S\rangle\}$

Equivalence ~~class~~ Relations p-218

যে relation টা reflexive, transitive ও symmetric হবে তাকে Equivalence relation বলে, যেমন,—

$A = \{1, 2, 3, 4\}$

$R = \{\langle 1,1\rangle, \langle 2,2\rangle, \langle 3,3\rangle, \langle 4,4\rangle, \langle 1,2\rangle, \langle 2,1\rangle, \langle 3,4\rangle, \langle 4,3\rangle\}$

Equivalence class

Equivalence ~~class~~ relations দ্বারা equivalence class তৈরী হয়েছে।

S equivalence class of x in class P হবে তাকে

সমতুল্য শ্রেণী [x] দ্বারা। [x] এর সদস্য S এর relation অনুযায়ী,

$(y) \{ y \in [x] \Leftrightarrow y \in P \; \& \; y S x \}$

S equivalence class generated by Ram in the class P is [R] এর অর্থ নিম্ন প্রকারে হবে

$(y)[y \in [R] \Leftrightarrow y \in P \; \& \; y S R]$

R_2 equivalence class generated by 1 in A is [1].

What R_2 equivalence class is [1] in A? $[1] = \{1, 2\}$

$\Downarrow$

[2] হবে A?

$[2] = \{2, 1\}$

এক্ষেত্রে দুটি class তৈরী হচ্ছে একটি হল $\{1,2\}$ এবং অন্যটি হল $\{3,4\}$