Kandi Raj College – Department of Mathematics – Internal Examination – 5th Semester – Honours course

Full Marks: CC – 5 = 10 ; CC – 6 = 10 ; CC – 7 = 10 ; SEC – 1 = 05 [for MATHEMATICS HONOURS STUDENTS]

Full Marks: HGE – 1 = 10 [Only for students OTHER THAN MATHEMATICS HONOURS]

For students of MATHEMATICS HONOURS:

Answer CC-5, CC-6, CC-7 & SEC-1 in separate answer scripts and upload them separately

	CC – 5	**10**
	Answer any 2 (TWO) questions:	
1.	A function $f: [0,1] \to \mathbb{R}$ be defined by, $f(x) = \begin{cases} x, & x \text{ is rational in } [0,1] \\ 1-x, & x \text{ is irrational in } [0,1] \end{cases}$ Show that f is continuous at $1/2$ and discontinuous at every other points in $[0,1]$.	**5**
2.	Use Lagrange's Mean Value theorem to prove that $\frac{2}{\pi} < \frac{\sin x}{x} < 1$ for $0 < x < \frac{\pi}{2}$.	**5**
3.(i)	Define an open set in a metric space (X, d).	**1+4**
(ii)	Prove that a subset U of a metric space (X, d) is open if and only if for each point $x \in U$, there is an open ball B (not necessarily x as its centre) such that $x \in B \subseteq U$.	

	CC – 6	**10**
1.	Let G be a multiplicative group and $S = \{x^2 : x \in G\}$. Examine if S is a subgroup of G. If not, then under what condition will it become a subgroup of G.	**2+1**
2.	If $(\mathbb{Q}, +)$ denote the additive group of rational numbers and $(\mathbb{Q}_{>0}, \times)$ denote the multiplicative group of positive rational numbers, then prove that $(\mathbb{Q}, +)$ and $(\mathbb{Q}_{>0}, \times)$ are not isomorphic to each other.	**3**
3.	If $G = \langle a \rangle$ and $\lvert a \rvert = 18$, find the inverse of a^9. Let G be a finite group and $a, b \in G$. Show that the elements ab and ba have the same order.	**1+3**

	CC – 7	**10**
1.	Derive Newton-Raphson formula and write down its merits and demerits	**5**
2.	State and prove the Lagrange's interpolation formula.	**5**

	SEC – 1	**05**
1.	If p and q are true and r and s are false, find the truth value of $\{\sim (p \wedge q) \vee r\} \vee \{([\sim p \vee q] \vee \sim r) \wedge s\}$	**2**
2.	Examine for tautology and contradiction: (i) $(p \wedge q) \to (p \to q)$; (ii) $(p \vee q) \wedge (\sim q) \wedge (\sim p)$	**2**
3.	Translate into symbolic form: *No student in the class can speak English or knows Hindi.*	**1**

For students of OTHER HONOURS:

	HGE – 1	**10**
	Answer any 2 (TWO) questions:	
1.(i)	Examine the nature of discontinuity of f at 1, where $f(x) = x - [x], 0 < x < 2$.	**2**
(ii)	Evaluate, $\lim_{x \to 0} (1+x)^{1/x}$	**3**
2.	Find the radius of curvature of the ellipse $x^2/a^2 + y^2/b^2 = 1$.	**5**
3.	If f is differentiable on $[0,1]$ show by Cauchy's mean value theorem that the equation $f(1) - f(0) = f'(x)/2x$ has at least one solution in $(0,1)$.	**5**