**Kandi Raj College – Department of Mathematics – Internal Examination – 5<sup>th</sup> Semester – Program course**

**Full Marks: DSE – T – 1 = 10   and  SEC – T – 3 = 05 [Only for students opting for Mathematics as SEC]**

**DSE – T – 1**                                                                                                                                     **10**
**Answer either Group A or Group B**

**Group – A [Matrices & Linear Algebra]**                                                                                      **10**
Answer any TWO questions:

1. Find the row-reduced echelon form of $A = \begin{bmatrix} 2 & -1 & 3 \\ 3 & 2 & 1 \\ 1 & -4 & 5 \end{bmatrix}$ and hence find its rank.          **05**

2. Find a basis for $\mathbb{R}^3$ that contains the vectors $(1,2,0)$ and $(1,3,1)$.          **05**

3. Prove that a linear transformation $T$ of a vector space $V$ of dimension $n$ to a vector space $V'$ of          **05**
   dimension $m$ over the same field $F$ is represented by an $m \times n$ matrix  over $F$ corresponding to
   the respective bases of $V$ and $V'$.

**Group – B [Complex Analysis]**                                                                                                       **10**

1. (i) Define an analytic function.          **1**

   (ii) Show that the function $f(z) = |z|^2, \forall z \in \mathbb{C}$, is differentiable at $(0,0)$ but not analytic at $(0,0)$.          **2**

2. Evaluate $\int_\gamma (e^z + z^2)/(z - 1)\, dz$ , where, $\gamma: |z| = 2$.          **3**

3. Find the radius of convergence of $\sum_{n=0}^\infty \{3 + (-1)^n\}^n\, z^n$.          **2**

4. Find the Laurent Series expansion of $f(z) = \frac{1}{z-2}$ in the region $|z| < 1$.          **2**

**Answer either Group A or Group B**

**Group – A [Integral Calculus]**                                                                                                       **05**

1. Using reduction formula $J_n = \int_0^{\pi/2} \cos^n x\, dx$, find $\int_0^{\pi/2} \cos^6 x\, dx$.          **3**

2. Evaluate $\iint_R \sin(x + y)\, dxdy$ over $R: \{0 \leq x \leq \frac{\pi}{2}, 0 \leq y \leq \frac{\pi}{2}\}$.          **2**

**Group – B [Vector Calculus]**                                                                                                         **05**

1. Find a vector of magnitude 5 perpendicular to both the vector  $2\hat{\imath} + \hat{\jmath} - 3\hat{k}$ and $\hat{\imath} - 2\hat{\jmath} + \hat{k}$.          **2¹/₂**

2. If, $\vec{A} = 2xz^2\hat{\imath} - yz\hat{\jmath} + 3xz^3\hat{k}$, find $\vec{\nabla} \times \vec{A}$ at the point $(1,1,1)$.          **2¹/₂**