2021
MATHEMATICS
[GENERAL]
Paper : IV

Full Marks : 100 Time : 3 Hours

The figures in the right-hand margin indicate marks.

1. Answer any **six** questions: 1×6=6

a) What is the position of C.G. of a square lamina?

b) State conservation of linear momentum.

c) In a central orbit prove that $r^2\dot{\theta}$ = constant.

d) What do you mean by apse in a central orbit?

e) What is escape velocity?

f) Find the sum of the approximated numbers 32.629, 72.3, 443.18695, 13.217 each being corrected to its last figure but no farther.

g) Write down Newton's forward interpolation formula.

h) Convert $(1011)_2$ to the equivalent decimal number.

i) Define parallelogram law of forces.

2. Answer any **eleven** questions: 2×11=22

a) State the forces which appear in the principal of virtual work.

b) State any two laws of friction.

c) A particle thrown vertically upward takes t seconds to rise to a height h and t′ seconds is the subsequent time to reach the ground again.

Show that $h = \frac{1}{2}gtt'$.

d) If radial velocity is proportional to the transverse velocity, find the path in polar coordinates.

e) Write down the differential equation of S.H.M. Hence write down the periodic time.

f) A gun of mass 1500 lbs fired a shot of 15 lbs with velocity 1000 ft/sec. How far the gun will recoil up a smooth inclined planes of inclination 1 in 8?

g) Find the horsepower of an engine draws a train at an uniform rate v ft/sec against a resistance R poundal.

h) Round up the following numbers correct upto six significant figures 24.564986, 27.483554.

i) From the following data form the difference table:

x	−1	0	1	2	3	4
f(x)	−2	1	0	1	10	33

j) What is the formula to compute the root of the equation $f(x)=0$ by Newton-Raphson method?

k) Write down Trapezoidal rule for integration.

l) Prove that $\Delta\nabla = \Delta - \nabla$ where Δ and ∇ have their usual meaning.

m) Multiply the numbers $(101.11)_2$ by $(11.101)_2$ and convert the result into decimal.

n) What is flow chart?

o) Three forces P, Q, R act along the line $x=0$, $y=0$ and $x\cos\theta + y\sin\theta = p$. Find the magnitude of resultant.

3. Answer **seven** questions: 6×7=42

a) Define centre of gravity. Find the centre of gravity of a semi-circular lamina.

b) A bullet of mass m moving with velocity v strikes a block of mass M which is free to move in the direction of motion of the bullet and is embedded in it. Show that a portion

$\frac{M}{M+n}$ of the K.E. is lost.

c) A heavy uniform chain, of length $2l$, hangs over a pulley, the length $l+c$ being at one side and $l-c$ at the other. Show that the chain will slip off the pulley in time $\left(\frac{l}{g}\right)^{\frac{1}{2}} \log\frac{l+\sqrt{l^2-c^2}}{c}$.

d) Find radial and cross radial components of velocity and acceleration.

e) A particle moves freely in a parabolic path given by $y^2 = 4ax$ under a force always perpendicular to its axis. Find the law of force.

f) Establish Newton's forward interpolation formula. When is this formula used?

g) Establish numerical differentiation formula based on either forward or backward formula.

h) Subtract: $(10101)_2 - (10001)_2$ by any method.

i) Convert the decimal number 17 and 37 into their binary equivalents and obtain their product using binary arithmatic.

j) State and prove virtual work for a system of coplanar force.

4. Answer any **three** questions: 10×3=30

a) i) Establish Lagrange's polynomial interpolation formula. If $x_0, x_1, ..., x_n$ are the interpolating points and $l_i(x)(i = 0, 1, ..., n)$ are the Lagrangian functions then show that $\sum_{i=0}^{n} l_i(x) = 1$.

ii) Describe the method of fixed point iteration to find the root of the equation $f(x) = 0$. 5+5

b) i) Show that any system of forces acting on a rigid body can be reduced to a single force together with a couple whose axis is along the direction of the force.

ii) A rough wire is in the form of an ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$. A small bead on the wire is in the limiting equilibrium when at a distance $\frac{b}{2}$ from the major axis under the action of a force whose line of action passes through the centre of the ellipse. Find the coefficient of friction. 5+5

c) i) Write a FORTRAN program to sum the series $y = x + \frac{x^3}{3} + \frac{x^5}{5} + ... + \frac{x^{15}}{15}$ when $x = 0.1, 0.3, 0.5, 0.7, 0.9$. The term having values less than 10^{-12} should not included in the sum.

ii) A Fibonnaci sequence is defined as follows 0, 1, 1, 2, 3, 5, ... $(T_{n+1} = T_n + T_{n-1},\ n \geq 1)$. Design an algorithm and draw a flow chart to find all numbers of this sequences which are ≤ 500. 5+5

d) i) Find the tangential and normal components of velocity and acceleration of a particle moving along a plane curve.

ii) Write down Kepler's laws of planetary motion. Also find the law of force when a particle describes the curve $p^2 = ar$ under a force F to the pole. 5+5

e) i) The horsepower required a steamer of mass M tones at its maximum speed V feet/sec. The resistance is proportional to the square of the speed and the engine exerts a constant

propeller thrust at all speeds. If in time t from rest the steamer acquired a velocity of v feet per second, then prove that

$$t = \frac{112}{55} \cdot \frac{MV^2}{Hg} \log_e \frac{V+v}{V-v}.$$

ii) If the displacement of a moving point at any time be given by a equation of the form $x = a\cos kt + b\sin kt$ then show that the point executes S.H.M. If $a = 3$, $b = 4$, $k = 2$, find the period. 5+5