2021
STATISTICS
[HONOURS]
Paper : VIII

Full Marks : 80 Time : 4 Hours

The figures in the right-hand margin indicate marks.
Candidates are required to give their answers in their own words as far as practicable.

Answer all the questions.

1. Answer any **seven** questions of the following: $1\times7=7$

   i) State the situation for use of Randomised Block Design (RBD).

   ii) State one advantage of Latin Square Design (LSD).

   iii) State one practical example where complete enumeration cannot be applied.

   iv) Give the expression of any one main effect of a 2^2-experiment.

[Turn over]

   v) Give a practical example of use of auxilliary information in ratio and regression methods of estimation.

   vi) Name a design in which all the basic principles of design are not used.

   vii) What is cost optimisation in sample survey?

   viii) Give a practical example of bias due to faulty sampling method.

   ix) If the observed F in Analysis of Variance comes out to be less than 1, what would be your conclusion?

   x) If an experiment is likely to produce lot of missing observations, which design would you recommend?

2. Answer any **six** questions of the following: $2\times6=12$

   i) Find the probability that a definite unit will be selected in a sample of size n by SRSWR, from a population of size N.

   ii) When do we use cluster sampling?

   iii) What do you mean by partially balanced Confounded Design?

   iv) Show that any treatment contrast may be written as a linear combination of the elementary treatment contrasts.

v) When do we use a split-block design?

vi) Distinguish between Analysis of Regression and Analysis of Covariance model.

vii) Distinguish between random effects and mixed effects model.

viii) Why is systematic sampling only partly probabilistic?

ix) What is proportional allocation? When do we use it?

3. Answer any **three** questions of the following: 7×3=21

i) Consider the multiple regression model

$$E(y_i) = \alpha + \beta_1 x_{1i} + \beta_2 x_{2i} + \ldots + \beta_p x_{pi};$$

$$i=1, 2, \ldots, n; \; n \gg p+1.$$

Derive the test for the hypotheses

$$H_{01}(\beta_1 = \beta_2 = \ldots = \beta_p = 0) \text{ and } H_{02}(\beta_p = 0).$$

ii) For n pairs of observations (x_i, y_i) using SRSWOR procedure from a population of size N, obtain the exact bias of the ratio estimator of population mean of y. Also find the mean square error of the ratio estimator.

iii) Describe two phase sampling procedure. Obtain the unbiased estimator of population mean of y in a double sampling regression procedure in large sample. Also obtain its mean square.

iv) Show that with n observable quantities we can form atmost (n–1) mutually orthogonal contrasts. Show that the sum of squares of full set of contrasts is the total sum of squares.

v) Show that for 2^n experiment, SS due to any effect $X = \frac{[X]^2}{2^n.r}$, where [X]=Factorical effect total of X and r is the number of replications. Use it to find the expression of SS(ABC) in 2^3 experiment.

4. Answer any **four** questions of the following: 10×4=40

i) Derive the sample sizes under

a) proportional allocation and

b) optimum allocation.

Show that if f.p.c. is ignored,

$$V_{opt.} \le V_{prop.} \le V_{ran.}.$$

ii) For a $(2^5, 2^2)$ design, some treatments in one of the blocks are given by a, b, bde, e .... Find the key block. Complete the other blocks. Also find the confounded effects.

iii) Describe the analysis of a strip-plot design with p levels of a factor A and q levels of another factor B arranged in r randomised blocks.

iv) Derive the analysis of a two way random effect model with equal number of observations per cell (more than one).

v) Consider an LSD with one missing observation. Derive the variances of the estimates of all elementary contrasts and hence obtain its efficiency with respect to the original design.

vi) For a SRSWOR of size n from a population of size N, obtain the unbiased estimator of the proportion with its variance. Also obtain the unbiased estimator of the variance.

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