U.G. 4th Semester Examination - 2021

MATHEMATICS

[PROGRAMME]

Course Code : MATH-G-CC-T-4

Full Marks : 30 Time : $1\frac{1}{2}$ Hour

The figures in the right-hand margin indicate marks.

The symbols and notations have their usual meanings.

1. Answer any **five** questions: 2×5=10

a) Find all the cyclic subgroups of the symmetric group S_3.

b) In a Ring R with unity 1 , prove that $(-1).a = -a$.

c) Give an example of an odd permutation and an even permutation.

d) Prove that the centre Z(G) of a group G is a normal subgroup of G.

e) Prove that two left Cosets a H, b H of H in G will be identical iff $a^{-1}b \in H$.

f) If H be a subgroup of a commutative group G, then show that the quotient group G/H is commutative.

g) Prove that every subring of the ring Z is an ideal.

h) Show that the set $\{a + b\sqrt{2}; a, b \in Q\}$ where Q is a set of Rational numbers, forms a group with respect to addition.

2. Answer any **two** questions : 5×2=10

a) Prove that every subgroup of a cyclic group is cyclic.

b) Show that the set of all permutations on the set {1,2,3} forms a non Abelian group.

c) Prove that intersection of two subrings is a ring. Is it true for union? 4+1

d) Let a, b ∈ G, such that o(a) =3 and $aba^{-1}=b^2$, then show that o(b)=7.

e) The set of all units in a ring R with unity forms a group with respect to multiplications.

3. Answer any **one** question : 10×1=10

a) i) Prove that a finite group of order n is cyclic, if it contains an element of order n.

ii) If a, b be two elements of a group (G, .) then show that a .x =b and y. a=b have unique solutions.

b) i) A subgroup H of a group G is normal iff $aHa^{-1}=H$ for every a in G.

ii) If (G, .) be a group such that for a, b $\in$ G, $a.b=b.a^{-1}$ and $b.a=a.b^{-1}$. Show that $a^4=b^4=e$.

c) i) Prove that every group of prime order is cyclic.

ii) Prove that the order of each element of a finite group is a divisor of the order of the group.