U.G. 2nd Semester Examination - 2021

MATHEMATICS

[HONOURS]

Course Code : MATH-H-CC-T-03

(Real Analysis)

Full Marks : 30 Time : $1\frac{1}{2}$ Hours

The figures in the right-hand margin indicate marks.

The symbols and notations have their usual meanings.

1. Answer any **five** questions: 2×5=10

a) If a is a non-zero real number, then show that $a^2 > 0$.

b) If *a* and *b* are two real numbers and $ab \geq 0$, then prove that $|a+b| = |a| + |b|$.

c) Let $S = \{1 - \frac{(-1)^n}{n} : n \in \mathbb{N}\}$. Find sup *S* and inf *S*.

d) Show that $\{r\in: r \in \mathbb{Q}\}$ is dense in $\mathbb{R}$.

e) Show that $\mathbb{N}$ is unbounded.

f) Show that if $a, b \in \mathbb{R}$ and $a \neq b$, then there exist ε-neighbourhoods *U* of *a* and *V* of *b* such that $U \cap V = \phi$.

g) If $a \in \mathbb{R}$ is such that $0 \leq a < \varepsilon$ for every $\varepsilon > 0$, then show that $a = 0$.

[Turn Over]

h) Let $S \subseteq \mathbb{R}$ and $S \neq \phi$ Show that *S* is bounded iff there exists a closed bounded interval *I* such that $S \subseteq I$.

2. Answer any **two** questions: 5×2=10

a) Show that $\lim\limits_{n\to\infty} \frac{1}{3^n} = 0$.

b) Show that unbounded sequences are not-convergent.

c) Let $x_n = (1+2^n)^{\frac{1}{n}}$. Show that $\lim\limits_{n\to\infty} x_n = 2$.

d) Let $x_1 = 2$ and $x_{n+1} = x_n + \frac{1}{x_n}$. Determine whether $\{x_n\}$ converges or diverges.

e) Let $|x_{n+1} - x_n| < \frac{1}{2^n}$ for all $n \in \mathbb{N}$. Show that $\{x_n\}$ is a Cauchy sequences.

3. Answer any **one** question: 10×1=10

a) Test whether the following series converges or diverges. 5+5

i) $\sum_{n=1}^{\infty} \frac{1}{n^{\sqrt{2}}}$.

ii) $\sum_{n=1}^{\infty} \frac{1}{n^{\frac{1}{2}}}$.

b) Show that following series are convergent. 5+5

i) $\sum_{n=1}^{\infty} \frac{1}{n(\ln n)^{\sqrt{2}}}$.

ii) $\sum_{n=1}^{\infty} \frac{\cos n}{n^2}$.

c) i) Test whether the series is convergent or divergent. 5+5

$$\sum_{n=1}^{\infty} \frac{(-1)^n}{n}$$

ii) Let $\{x_n\}$ be real sequences such that $x_n \leq \frac{1}{n^2+n}$ for all $n \in \mathbb{N}$. Test the series $\sum_{n=1}^{\infty} x_n$ whether convergent or divergent.
