U.G. 2nd Semester Examination - 2021

MATHEMATICS

[HONOURS]

Course Code : MATH-H-CC-T-04

(Differental Equations & Vector Calculus)

Full Marks : 30 Time : $1\frac{1}{2}$ Hours

The figures in the right-hand margin indicate marks.

The symbols and notations have their usual meanings.

1. Answer any **five** questions: 2×5=10

a) Solve: $\frac{dy}{dx}=\frac{3x-4y-2}{6x-8y-5}$

b) Find the unit vector which is perpendicular to the vectors $3\vec{i}-2\vec{j}+\vec{k}$ *and* $2\vec{i}-\vec{j}-3\vec{k}$.

c) Verify whether $\frac{1}{(x+y+1)^4}$ is an integrating factor of $(2xy-y^2-y)dx+(2xy-x^2-x)dy=0$.

d) Solve the equation $x^2p^2-2xyp+y^2=x^2y^2+x^4$.

e) Reduce the equation $x^2p^2+py(2x+y)+y^2=0$ to Clairaut's form.

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f) Find the directional derivative of $\varphi(x,y,z)=2yz+x^2$ at the point (1,1,2) in the direction of the vector $\vec{i}+\vec{j}+2\vec{k}$.

g) Integrate $a\times\frac{d^2\vec{r}}{dt^2}=b$, where a and b are constant vectors.

h) Give geometrical interpretation of $\frac{d\vec{r}}{dt}$.

2. Answer any **two** questions: 5×2=10

a) If $\vec{\alpha}=t^2\vec{i}-t\vec{j}+(2t-1)\vec{k}$ and $\vec{\beta}=(2t-3)\vec{i}-\vec{j}-t\vec{k}$, find $\frac{d}{dt}\left(\vec{\alpha}\times\frac{d\vec{\beta}}{dt}\right)$ at $t=2$.

b) Find the general solution of

$$(2x+1)(x+1)\frac{d^2y}{dx^2}+2x\frac{dy}{dx}-2y=(2x+1)^2.$$

c) Solve:

$$(5+2x)^2\frac{d^2y}{dx^2}-6(5+2x)\frac{dy}{dx}+8y=8(5+2x)^2.$$

d) Solve: $\sin x\frac{d^2y}{dx^2}-\cos x\frac{dy}{dx}+2y\sin x=0.$

e) A particle moves along the curve $x=t^3+1, y=t^2, z=2t+5$, where t is the time. Find the components of its velocity and acceleration at t=1 in the direction of $\vec{i}+\vec{j}+3\vec{k}$.

3. Answer any **one** question: 10×1=10

a) i) Solve: $x^2\frac{d^2y}{dx^2}+3x\frac{dy}{dx}+y=\frac{1}{(1-x)^2}$.

ii) Find the unit tangent vector at any point on the curve

$x=a\cos t, y=a\sin t, z=bt.$ 5+5

b) i) Verify whether the equation

$$(2x^2+3x)\frac{d^2y}{dx^2}+(6x+3)\frac{dy}{dx}+2y=(x+1)e^x$$

is exact and then solve it.

ii) Given

$\vec{F}=(3x-2y)\vec{i}+(y+2z)\vec{j}+x\overrightarrow{k},$

evaluate $\int_C \vec{F}.d\vec{r}$ from (0,0,1) to (1,1,1) considering the curves

C1: $x=t,\ y=t^2,\ z=t^3$ and

C2: $x=z^2, z=y^2$. 5+5

c) i) Solve the system: $(D^2-2)x-3y=e^{2t}$, $(D^2+2)y+x=0$.

ii) Given that $\vec{r}(t)=2\vec{i}-\vec{j}+2\vec{k}$, when t=2 and $\vec{r}(t)=4\vec{i}-2\vec{j}+3\vec{k}$, when t=3. Show that $\int_2^3(\vec{r}.\frac{d\vec{r}}{dt})\,dt=10$. 5+5

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