2021
STATISTICS
[HONOURS]
Paper : I

Full Marks : 75 Time : 4 Hours

The figures in the right-hand margin indicate marks. Candidates are required to give their answers in their own words as far as practicable.

Answer all the questions.

GROUP–A
(Probability–I)
(Marks : 45)

1. Write down whether the following statements are "**true**" or "**false**" (any **three**): 1×3=3

   a) For two random variables X and Y if $E(XY)=E(X)E(Y)$, then X and Y are independent.

   b) Correlation coefficient ρ between two random variables X and Y is ± 1 if and only if there exists constants a and b such that $P[y=ax+b]=1$.

   c) Suppose X has a geometric distribution. Then $P\{X>m+n \mid X>m\}=P\{X\ge n\}$ for any two positive integers m and n.

   d) For any three events A, B, C, $P(A\cap B\cap C)=P(A).P(B\mid A)P(C\mid AB)$

   e) Two mutually exclusive events are always independent.

2. Answer any **five** questions: 2×5=10

   a) For the truncated Poisson distribution

   $$f(x)=\frac{e^{-\lambda}\lambda^{x}}{x!\left(1-e^{-\lambda}\right)}, x=1,2,3,...$$

   find the moment generating function. 2

   b) For the standard normal density $\varphi(x)$, show that $\varphi'(x)+x\varphi(x)=0$. 2

   c) Find the distribution function of a random variable X having the Cauchy distribution with parameters μ and σ. 2

   d) Explain the statement 'probability of an event is its long-run relative frequency'. 2

   e) Give an example such that $P(A\cup B)<P(A)+P(B)$. 2

   f) A town contains 4 people that repair television.

If 4 sets break down, what is the probability that at least 1 of repairers is called? 2

g) If X is a geometric random variable, show that $P\{X=n+k \mid X>n\}=P(X=k)$. 2

3. Answer any **two** questions: 6×2=12

a) State and prove Poincares' theorem.

b) Define a quartile based measure of dispersion and find such measure for a random variable with pdf

$$f(x)=\frac{1}{\sigma}e^{-x/\sigma},\ x\geq 0,\ \sigma>0.$$

c) A die is thrown as long as necessary for a 6 to turn up. Given that the '6' does not turn up at the first throw, what is the probability that more than four throws will be necessary?

4. Answer any **two** questions: 10×2=20

a) i) A fair coin is tossed repeatedly. Suppose that heads appears for the first time after X tosses and tails appears for the first time after Y tosses. Find the joint distribution and the marginal distributions of X and Y.

ii) Show that, $E(XY)=E(X).E(Y)$ if X and Y are two independent random variables. 5+5

b) Derive the binomial distribution from a suitable probability model. Examine the skewness of the distribution. 4+6

c) i) If $A_1, A_2, A_3, \ldots A_n$ are n not necessarily mutually exclusive events find $P\left(\bigcup_{1}^{n} A_i\right)$.

ii) Obtain the probability that in k throws of a die each of the numbers 1, 2, ..6 will appear at least once. 5+5

GROUP - B

(Mathematical Methods-I)

(Marks : 30)

5. Write down whether the following statements are "**true**" ot "**false**" (any **two**): 1×2=2

a) If A is a symmetric matrix then the eigenvalues of A are real.

b) If $f(x)=[2x]$, then $\lim_{x\to 1/2} f(x)=1$.

c) Union of two vector spaces is always a vector space.

6. Answer any **one** question: 2×1=2

a) Consider that the m × n matrix E_{mn} is defined as a matrix all of whose elements have the value unity. What is the matrix $E_{nm}A$ for any m×r matrix A? 2

b) Indicate the use of Gram-Schmidt orthogonalization process. 2

7. Answer any **one** question: 6×1=6

a) Show that a necessary and sufficient condition for a quadratic form $\underset{\sim}{x}'A\underset{\sim}{x}$ to be positive definite is that all the eigen values of A satisfy $\lambda i > 0$. In that case show that all principal order minors are positive.

b) Let A and B be the two matrices such that the product AB is defined. Then prove that Rank (AB) $\leq$ min {Rank(A), Rank(B)}.

8. Answer any **two** questions: 10×2=20

a) i) Let $S = \left\{ \underset{\sim}{x} : \underset{\sim}{x} = (x_1, x_2, 0)', x_1, x_2 \in \quad \right\}$.

Find a basis for S and its dimension.

ii) Discuss the usefulness of partitioning of matrices in matrix multiplication with an example.

iii) Evaluate $\lim\limits_{x \to 0} \frac{xe^x - \log(1+x)}{x^2}$

4+4+2

b) i) Examine the function

$f(x) = (x-3)^4 (x+1)^3$

for extreme values.

ii) Suppose P and Q are two matrices of order n×n such that PQ=0. Show that column space (Q) $\subset$ Null space (P) and hence Rank (P) + Rank (Q) $\leq$ n. 4+6

c) i) A function f is defined on $\mathbb{R}$ by

$$f(x) = \begin{cases} -x^2 & \text{if } x \leq 0 \\ 5x - 4 & \text{if } 0<x \leq 1 \\ 4x^2 - 3x & \text{if } 1<x<2 \\ 3x + 4 & \text{if } x \geq 2 \end{cases}$$

Examine f for continuity at x=0, 1,2. Also discuss the kind of continuity.

ii) Discuss how the basis of a vector space cannot be unique. 5+5

———————