2021
STATISTICS
[GENERAL]
Paper : II

Full Marks : 100 Time : 3 Hours

The figures in the right-hand margin indicate marks.

Candidates are required to give their answers in their own words as far as practicable.

Answer all the questions.

1. Answer any **six** questions: 1×6=6

   a) Write the fixed effects model for ANOVA one-way classified data.

   b) Define power in the context of testing of hypothesis.

   c) What is marginal distribution?

   d) Write down one layout of 4×4 LSD.

   e) Define a unbiased estimator.

   f) Write down the pdf of a F-distribution with n_1 and n_2 degrees of freedom.

   g) What is null hypothesis?

   h) What is non sampling error in sample survey?

2. Answer any **eleven** questions: 2×11=22

   a) State 'Weak law of large numbers'.

   b) Obtain the total number of possible samples of size n drawn with replacement from a finite population of size N.

   c) What are confidence limit and confidence coefficient? Explain clearly.

   d) Write down the properties of a t-distribution.

   e) What do you mean by MVUE?

   f) Write down the expression for the standard error of sample mean in SRSWOR from a finite population.

   g) What are the different models in ANOVA?

   h) Give an example, where maximum likelihood estimator is biased.

   i) Write down the model suitable for analysis of data obtained from an RBD with proper assumption.

   j) If $\bar{X}$ is an u. e. of a population mean μ, show that $\bar{X}^2$ is, in general biased for μ^2.

   k) Distinguish between a null hypothesis and an alternative hypothesis with examples.

   l) Explain what do you mean by local control in connection with design of experiments.

m) State Chebyshev's inequality.

n) If x_1, x_2, x_3, x_4 are independently distributed N(0, 1) variables, what will be the distribution of $(x_1^2 + x_3^2)/(x_2^2 + x_4^2)$?

3. Answer any **seven** questions: 6×7=42

a) In connection with sample survey, explain the terms population, sample, sampling unit and sampling frame.

b) Describe the model, the assumptions and the hypothesis for analysing two way classified data with one observations per cell under fixed effects model.

c) Derive the maximum likelihood estimate of the parameters α of a population having density function

$$f(x) = \begin{cases} \frac{2}{\alpha^2}(\propto - x), & 0 < x < \infty \\ 0, & \text{otherwise} \end{cases}$$

for a sample of unit size.

d) Let x be a r.v. with pdf

$$f(x) = \begin{cases} 1, & \theta - \frac{1}{2} < x < \theta + \frac{1}{2} \\ 0, & \text{O.W.} \end{cases}$$

To test $H_0 : \theta = 2$ against $H_0 : \theta = 4$, suppose we reject H_0 if $x > 3$. Find P(Type I error) and P(Type II error) of the test.

e) The joint probability density function of r.v. X and Y is given by

$$f(x, y) = \begin{cases} \frac{6}{7}\left(x^2 + \frac{xy}{2}\right), & 0 < x < 1, \ 0 < y < 2 \\ 0, & \text{otherwise} \end{cases}$$

I) verify that this is indeed a joint density function.

II) Compute the marginal function of x.

III) Find E (X).

f) If x_1,x_2, are independently distribute random variables with N(0,1) derive the distribution of x_1+x_2.

g) Discuss a suitable test for testing $H_0 : \mu_1 = \mu_2$ against all alternatives for two independent normal populations $N(\mu_1, 1)$ and $N(\mu_2, 2)$.

h) Let $x_1, x_2, \ldots, x_n$ are n random sample observations from $N(\mu, a^2)$. Obtain $100(1-\alpha)\%$ confidence limits to a^2, when μ is known.

i) What are the basic principles of design of experiment? Explain them clearly.

4. Answer any **three** questions: 10×3=30

a) For a normal distribution $N(\mu, \sigma^2)$, both μ and σ being unknown, discuss testing procedures for

i) $H_0 : \mu = \mu_0$ against $H_1 : \mu \neq \mu_0$ and

ii) $H_0 : \sigma = \sigma_0$ against $H_1 : \sigma < \sigma_0$

Also derive a 95% confidence interval for μ. 3+3+4

b) i) Write down the role of randomization, replication and local control in design of experiments.

ii) Derive the variance of sample mean in case of SRSWOR from a finite population. 5+5

c) Write notes on: 5+5

i) Method of moments

ii) Properties of ML estimator.

d) Obtain the mean and variance of chi-square distribution with m degrees of freedom. 5+5

e) If $x_1, x_2, \ldots, x_n$ are n random sample obseravtions from $N(\mu, a^2)$. Examine the unbiasedness of the sample mean as an estimator of μ. Derive the MLE of μ and a^2. 4+6

_____