QUESTION PAPER FOR MATHEMATICS HONOURS STUDENTS : Marks distribution: 10+10+10+5
Answer Each Paper In Separate Answer Script And Submit Separately

PART – 1: CC5   **10**
Answer any TWO questions: 2 × 5

1. A function f: $[0, 1] \to R$ is defined by
    $$f(x) = \begin{cases} x, & x \text{ is rational in } [0, 1] \\ 1 - x, & x \text{ is irrational in } [0, 1] \end{cases}$$
    Show that f is continuous at ½ and discontinuous at every other point in [0, 1].

2. State and prove Cauchy's Mean Value theorem.

3. Let $c \in R$ and a real function f be such that f" is continuous on some neighborhood of c.
    Prove that $\lim\limits_{h \to 0} \dfrac{f(c+h) - 2f(c) + f(c-h)}{h^2} = f''(c)$

4.(i) Show that the intersection of a finite family of open sets in a metric space (X, d) is open.

 (ii) Show by an example that, the above result is not true for the intersection of infinite family of open
       sets.

PART – 2: CC6   **10**
Answer any ONE question: 1 × 10

1. Suppose $p > 3$ is a prime and also assume that $G = U_p$.   2+4+4
    Find $|G|$. Show that $H = \{x^2 : x \in G\}$ is a subgroup of the multiplicative group G.
    Determine the index $[G:H]$.

2.(i) Let G be a group and $H \leq G$ and $N \Delta G$.   5
       Show that $HN = \{hn \in G : h \in H, n \in N\}$ is a subgroup of G.

 (ii) Let G be a finite group and $S = \{x \in G : x^5 = e\}$, where e is the identity of G. Prove that the   5
       number of elements in S is odd.

PART – 3: CC7   **10**
Answer any TWO questions: 2 × 5

1. State and prove Lagrange's Interpolation formula.   5

2. Derive Newton-Raphson formula and show that it has a quadratic convergence.   5

PART – 4:SEC   **05**
Answer any ONE question: 1 × 5

1. What is a conditional and a bi-conditional connective?   2+3
    Suppose the statement "I wear my running shoes if and only if I am exercising" is true. Determine
    if the following statements are "TRUE" or "FALSE".
    (a) I am exercising and I am not wearing my running shoes;
    (b) I am wearing my running shoes and I am not exercising;
    (c) I am not exercising and I am not wearing my running shoes;

2. What is a Tautology?   1+4
    Examine using truth-table if the following argument is a Tautology or not:
    "If you bought bread, then you went to the store. And you bought bread. Then it implies that you
    went to the store."

QUESTION PAPER FOR MATHEMATICS HONOURS STUDENTS ENDS HERE

QUESTION PAPER FOR OTHER HONOURS STUDENTS : Marks distribution: 10

HGE   **10**
Answer any TWO questions: 2 × 5

1. If y = tan^{-1}x, show that $(1 + x^2)y_{n+2} + 2(n + 1)xy_{n+1} + n(n+1)y_n = 0$

2. Prove that the pedal equation of the astroid , $x^{2/3} + y^{2/3} = a^{2/3}$ (a>0) is $r^2 + 3p^2 = a^2$.

3. Prove that between any two real roots of e^{x}sin x = 1, $\exists$ at least one real root of e^{x}cos x +1 = 0.