## U.G. 3rd Semester Examination - 2021

# PHYSICS

### [HONOURS]

### Course Code : PHY-H-CC-T-05

### (Mathematical Physics-II)

Full Marks : 40 Time : $2\frac{1}{2}$ Hours

*The figures in the right-hand margin indicate marks.*

*Candidates are required to give their answers in their own words as far as practicable.*

1. Answer any **five** questions: 2×5=10

a) Solve the differential equation by Frobenius method

$$y''=0\left(\text{where } y''=\frac{d^2y}{dx^2}\right).$$

b) Find out the regular singular point of the given differential equation.

$$\left(1-x^2\right)y''+2x^2y'+(x-2)y=0.$$

c) Let y = ab. Find out the expression for the maximum permissible error in y.



d) Evaluate : $\left|\!\overline{\left(\frac{5}{2}\right)}\right.$

e) Find out $\int_{-1}^{1}x^2P_2(x)dx$ where $P_2(x)$ is the 2nd Legendre Polynomial.

f) Find out $a_o$ of the function $f(x)=\frac{1}{4}(\pi-x)^2$, $a_o$ is the Fourier series coefficient.

g) Evaluate the following integration $\int_0^{\infty}x^7e^{-x}dx$.

2. Answer any **two** questions: 5×2=10

a) Find out the Fourier series of the function f(x) given by

$$f(x)=\begin{cases} a & 0\le x\le\pi \\ 2\pi-x & \pi\le x\le 2\pi \end{cases}$$

and find the sum of $\sum_{n=0}^{\infty}\frac{1}{(2n+1)^2}$.

b) Solve the differential equation by Frobenius method

$$x^2y''+xy'+y=0.$$

c) State and prove Rodrigue's Formula for Legendre polynomials.

d) Evaluate the integration $\int_0^\infty \frac{e^{-k^2/\sigma^2}}{\sigma^6}\, d\sigma, k \neq 0$.

3. Answer any **two** questions: 10×2=20

a) i) Evaluate the integration

$$I_n = \int_0^1 \left(1-\sqrt{x}\right)^n dx.$$

ii) The specific resistance $\sigma$ of a thin wire of radius r cm, resistance R $\Omega$ and length L cm given by $\sigma = \frac{\pi r^2 R}{L}$.

If $r = 0.26 \pm 0.02\,\text{cm}$

$R = 32 \pm 1\Omega$

$L = 78 \pm 0.01\,\text{cm}$

Find the percentage error is $\sigma$.

iii) For the given periodic function

$$f(t) = \begin{cases} 2t & \text{for } 0 \le t \le 2 \\ 4 & \text{for } 2 \le t \le 6 \end{cases}$$

with a period T=6. Find out the Fourier coefficient $a_1$. 4+4+2

b) i) Show that the legendre polynomial are generated by the function.

$$g(x,t) = \frac{1}{\sqrt{1-2xt+t^2}}.$$

ii) Show that $\frac{d}{dx}\left[x^{-n} J_n(x)\right] = -x^{-n} J_{n+1}(x)$ where $J_n(x)$ is the Bessel's Function given by

$$J_n(x) = \sum_{r=0}^{\infty} (-1)^r \left(\frac{x}{2}\right)^{n+2r} \frac{1}{r!\,\lceil(n+r+1)}.$$

iii) Show that $J_n'(x) = J_{n-1}(x) - \frac{n}{x} J_n(x)$ where $J_n'(x) = \frac{d}{dx}\left(J_n(x)\right)$. 5+3+2

c) i) Let $i^2 = -1$ and suppose that u(x, y) and v(x, y) are such that

$$(x+iy)^4 = u(x,y) + iv(x,y).$$

Find u and v and show that both satisfy Laplace's equation that is

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0 \text{ and } \frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} = 0$$

in addition, show that u and v satisfy the condition given below

$u_x = v_y$ and $u_y = -v_x$

ii) Find out the partial differential equation of $u = f\left(x^2 - y^2\right)$.

iii) Solve $u_{tt} = c^2 u_{xx}, u(x,0) = e^x$

$u_t(x,0) = \sin x$

5+2+3

d) i) Solve the equation

$$u_x + u_y + u = e^{x+2y}$$

with $u(x,0) = 0$

ii) Show that

$$\int_0^{\frac{\pi}{2}} (\cos\theta)^{2k+1} d\theta = \frac{(k!)^2 2^{2k}}{(2k+1)!}$$

iii) For n=0, 1, 2, ...

$$\int_{-1}^{1} P_n^2(x) = \frac{2}{2n+1}$$

when $P_n(x)$ is the legendre polynomial.

iv) What is propagation of errors?

3+3+3+1

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