U.G. 3rd Semester Examination - 2021

MATHEMATICS

[HONOURS]

Generic Elective Course (GE)

Course Code : MATH-H-GE-T-03

Full Marks : 60 Time : $2\frac{1}{2}$ Hours

The figures in the right-hand margin indicate marks.

Notations & Symbols have their usual meanings.

1. Answer any **ten** questions: 2×10=20

a) Show that $\lim_{x\to 0}\frac{1}{2+e^{1/x}}$ does not exist.

b) What do you mean by non-removable discontinuity? Give an example of it.

c) If $2x = e^y + e^{-y}$, then express y explicitly in terms of x.

d) Evaluate $\lim_{x\to 0}\left[\frac{x-1+\cos x}{x}\right]^{1/x}$.

e) Show that $f(x)=|x|$ is not derivable at $x=0$.

f) If $f(x+y)=f(x).f(y)\ \forall x, y\in R$; $f(x)\neq 0$ for any real x and $f'(0)=2$, then show that $f'(x)=2f(x)$.

g) Show that the following pair of curves $r^2\theta = a^2$ and $r=e^{\theta^2}$ cut orthogonally.

h) State the Rolle's theorem with an example.

i) Find the maximum and the minimum values $f(x)=4-|\cos 3x|$.

j) Find the centre of curvature of the curve $xy = x^2+4$ at (2, 4).

k) Find the asymptotes of the curve $y^2(x-1)=x^3$.

l) Find the nature and position of the singular points of the curve $x^3-x^2y+y^2=0$.

m) If $y=\sin^2 x\cos^2 x$, find y_n, where $y_n\equiv\frac{d^n y}{dx^n}$.

n) Show that $x>\log(1+x)>x-\frac{x^2}{2}$; $(x>0)$.

o) If $u = x\log y$, then show that $\frac{\partial^2 u}{\partial x\partial y}=\frac{\partial^2 u}{\partial y\partial x}$.

2. Answer any **four** questions: 5×4=20

a) Prove that if ρ_1, ρ_2 are radii of curvatures of a focal chord of a parabola, whose semi-latus rectum is l, then $(\rho_1)^{-2/3}+(\rho_2)^{-2/3}=l^{-2/3}$.

b) Find the rectilinear asymptotes of the following curve:

$$x^3+x^2y-xy^2-y^3+2xy+2y^2-3x+y=0.$$

c) If $lx+my=1$ touches the curve $(ax)^m+(by)^n=1$, then show that $\left(\frac{l}{a}\right)^{n/(n-1)}+\left(\frac{m}{b}\right)^{n/(n-1)}=1$.

d) State and prove Lagrange's mean value theorem.

e) Show that $\lim_{x\to 0}\left(\frac{\tan x}{x}\right)^{1/x^2}=e^{1/3}$.

f) If $y=x^{2n}$, where n is the positive integer, then show that $y_n=2^n\left[1.3.5...(2n-1)\right]x^n$.

g) Find the second degree Taylor polynomial for $\cos x$ around $x=\pi$.

3. Answer any **two** questions: 10×2=20

a) i) Verify the Rolle's theorem for the function; $f(x)=x^2-5x+6$ in $1\le x\le 4$. 4

ii) If $u=\sin^{-1}\frac{x+y}{\sqrt{x}+\sqrt{y}}$, then find the value of

$$x^2\frac{\partial^2u}{\partial u^2}+2xy\frac{\partial^2u}{\partial x\partial y}+y^2\frac{\partial^2u}{\partial y^2}.$$ 6

b) i) State and prove Taylor's theorem with Cauchy's form of remainder. 5

ii) Trace the following curve:

$$y^2(x+3a)=x(x-a)(x-2a).$$ 5

c) i) If $u=r^3$, $x^2+y^2+z^2=r^2$, then show that

$$\frac{\partial^2u}{\partial x^2}+\frac{\partial^2u}{\partial y^2}+\frac{\partial^2u}{\partial z^2}=12r.$$ 5

ii) Find minimum value of $x^2+y^2+z^2$ subject to condition $xyz=a^3$. 5

d) i) Determine the existence and nature of double point on the curve $(x-2)^2=y(y-1)^2$. 6

ii) A function f is defined on R^2 by

$$f(x,y)=\begin{cases}\dfrac{xy^2}{x^2+y^4}, & (x,y)\neq(0,0)\\ 0, & (x,y)=(0,0).\end{cases}$$

Show that f is not continuous at (0, 0). 4

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