2022
MATHEMATICS
[HONOURS]
Paper : V

Full Marks : 100 Time : 4 Hours

The figures in the right-hand margin indicate marks.
Candidates are required to give their answers in their own words as far as practicable.
Symbols and notations have their usual meanings.

1. Answer any **five** questions: 1×5=5

   a) Define centre of pressure of a plane lamina.

   b) Define momentum ellipsoid at any point of a rigid body.

   c) Define cone of friction.

   d) What do you mean by field of force?

   e) What is metacentre?

   f) Define length of simple equivalent pendulum.

   g) State Lami's theorem.

   h) What is conservative field of force?

2. Answer any **ten** questions: 2×10=20

   a) A triangle is immersed in a homogeneous liquid. Show that the sum of pressure at the vertices is three times the pressure at the centre.

   b) A circular plane is immersed with its plane vertical in a liquid. Find the depth of its centre of pressure.

   c) Find the time period of oscillation of a compound pendulum.

   d) Find the moment of intertia of a hollow sphere about a diameter, its interval and external radii are a and b respectively.

   e) If a system consist of two forces, one of which act along OZ and mix components of the system are X, Y, Z; L, M, N, then show that the force along OZ is $\frac{LX + MY + NZ}{N}$.

   f) A homogeneous fluid is in equilibrium under gravity only. Show that the surface of equal pressure are horizontal planes.

   g) Explain D'Alembert's principle.

   h) Write the equations of motion of a rigid body moving in two dimension under impulsive forces.

   i) Prove that for a reversible adiabatic change $PV^{\gamma} = \text{constant}$. (Notations are usual)

   j) Prove that in a homogeneous fluid rest under gravity, the pressure difference between two points is proportional to the difference of their depths.

   k) State the principle of virtual work.

   l) Find the distance of C.G. of a sector of a circle from its centre.

3. Answer any **five** questions: 6×5=30

a) A liquid filled the lower half of a circular tube of radius a in a vertical plane. If the tube is now rotated about the vertical diameter with uniform angular velocity ω such that the liquid is just separate into two parts, show that $\omega = \sqrt{\frac{2g}{a}}$.

b) Forces P, Q, R act along the straight lines y=b, z = –c; z = c, x = –a; x = a, y = –b respectively. Show that they will have a single resultant if $\frac{a}{P}+\frac{b}{Q}+\frac{c}{R}=O$. Also find the equation of the line of single resultant.

c) An imperfectly rough sphere moves from rest down a plane inclined at an angle α to the horizon. Discuss the motion.

d) Establish the energy test of stability of equilibrium of a rigid body and explain it for one degree for freedom.

e) A quadrant of a circle is just immersed vertically in a liquid with one edge in the free surface. Find the position of the centre of pressure if the density of the liquid varies as the depth.

f) A uniform rod of length 2a, is placed on a rough table at right to its edge. If its centre of gravity be initially at distance 'b' beyond the edge, show that the rod will begin to slide when it has turned through an angle $\tan^{-1}\left(\frac{\mu a^2}{a^2+9b^2}\right)$, where μ is coefficient of friction.

g) A heavy uniform rod of length 2a and mass m can turn freely about one end which is fixed. If it starts with angular velocity ω from the position in which it hangs vertically, find the angular velocity. Also prove that the time of describing an angle θ is $2\sqrt{\frac{a}{3g}}\log\tan\left(\frac{\pi}{4}+\frac{\theta}{2}\right)$.

h) Investigate the condition of equilibrium of a particle constrained to rest on a rough plane curve $f(x, y)=0$ under any given forces in the plane of the curve.

Answer any **three** questions: 15×3=45

4. a) Establish the principle of independence of motion due to translation and rotation of a rigid body. 7

b) A rod of length 2a is suspended by a string of length l, attached to one end. It the string and the rod revolve with uniform angular velocity about a vertical and their inclination with the vertical be θ and φ respectively, then prove that $\frac{3l}{a}\frac{(4\tan\theta-3\tan\varphi)\sin\varphi}{(\tan\varphi-\tan\theta)\sin\theta}$. 8

5. a) A fine glass tube in the shape of an equilateral triangle is filled with equal volumes of two

liquids which do not mix, whose densities are in A.P. The tube is held in vertical plane and the side that contain the heaviest and lightest liquids makes angle θ with the verticle. Show that the surface of separation divide the sides the ratio $\cos\left(\frac{\pi}{6}-\theta\right):\cos\left(\frac{\pi}{6}+\theta\right)$. 7

b) A spherical vessel is just filled with a heavy liquid, the particle of which attract one another according to the law of gravitation. If the pressure at the highest point vanishes, show that the resultant thrust across a vertical plane is $\pi g\rho a^3+\frac{1}{3}\pi^2\rho^2\gamma a^4$ where a is the radius, ρ is the density and γ is the constant of gravitation. 8

6. a) Two uniform similar rods (of same material) PQ and QT of length $2l$ and $2l'$ respectively– are rigidly united at Q and suspended from P. If they inclined at an angle α and β respectively to the vertical, prove that $\left(l^2+2ll'\right)\sin\alpha=l'\sin\beta$. 7

b) Two equal forces act along the generations of the system of the hyperboloid $\frac{x^2+y^2}{a^2}-\frac{z^2}{b^2}=1$, and cut the plane z = 0 at the extremities of perpendicular diameters of the circle $x^2+y^2=a^2$. Show that the pitch of the

equivalent wrench is $\frac{a^2b}{a^2+2b^2}$. 8

7. a) Define principal axis. Obtain the conditions for a given straight line to be a principal axis of the system of some point of its length. If so, find the other two principal axes.

b) A non-homogeneous rod AB of length $2l$, whose density at any point is directly proportional to the distance of the point from A, is rotating with a uniform angular velocity ω about a vertical axis through A. If the rod incliend at an angle α to the vertical, show that the value of α is either 0 or $\cos^{-1}\left(\frac{2g}{3l\omega^2}\right)$. 7+8

8. a) A force parallel to z-axis acts at the point $(a, 0, 0)$ and an equal force perpendicular to z-axis acts at the point $(-1, 0, 0)$. Show that the central axis of the system lies on the surface $z^2\left(x^2+y^2\right)=\left(x^2+y^2-ax\right)^2$.

b) A quadrilateral ABCD formed of four uniform rods freely jointed to each other at their ends, the rods AB, AD being equal and also the rods BC, CD are freely suspended from the point A. A string joins A to C and such that $\angle$ABC is a right angle. Prove by using principle of

virtual work that the tension in the string is $(\omega+\omega')\sin^2\theta+\omega'$, where ω is the weight of the upper rod and ω' of a lower rod and $2\theta=\angle BAD$. 7+8

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