2022
PHYSICS
[HONOURS]
Paper : VIII

Full Marks : 80 Time : 4 Hours

The figures in the right-hand margin indicate marks.
Candidates are required to give their answers in their own words as far as practicable.

GROUP-A

1. Answer any **seven** questions: 1×7=7

a) Write down the expression of de Broglie wavelength of a particle with kinetic energy T.

b) In quantum mechanics, determine the dimension of the wavefunction $\psi(x, y, z)$ of a particle.

c) What do you mean by cyclic coordinate?

d) Three spin-$\frac{1}{2}$ non-interacting electrons are placed in a simple harmonic oscillator potential $\frac{1}{2}m\omega^2x^2$. Calculate the lowest allowed energy of the system.

e) Give an expression of the wave function in the momentum space which satisfies Scroedinger wave equation.

f) What is 'Bohr Magneton'? Give its value.

g) Show that the phase-space trajectory of a one-dimensional simple harmonic oscillator is an ellipse.

h) Mention the significance of Liouville's theorem.

i) What are configuration space and phase space variables?

GROUP-B

2. Answer any **six** questions: 2×6=12

a) Calculate the permitted energy levels of an electron in a one-dimensional box of width 1Å.

b) Both Lagrangian and Hamiltonian have the dimension of energy but Hamiltonian is not equal to the total energy in all situation. – Explain.

c) Which value of f(p) makes the transformations $Q = \frac{f(p)}{m\omega}\sin q;\ P = f(p)\cos q$ canonical?

d) Establish Rayleigh-Jeans law from Planck's radiation formula in the large wavelength limit.

e) Derive an expression of Planck's constant in terms of stopping potentials of a metal, V_{s1} and V_{s2}, corresponding to two different wavelengths λ_1 and λ_2 respectively.

f) In the hydrogen spectrum, find the ratio of the wavelengths for Lyman-α radiation (n=2 to n=1 transition) to Balmer-α radiation (n=3 to n=2 transition).

g) Prove D'Alembert's principle.

h) Explain the two major problems associated with constraints which compelled to modify Newton's law.

GROUP-C

3. Answer any **three** questions: 7×3=21

a) i) State and explain the principle of virtual work. Write down the advantages of this principle to solve a dynamical system.

ii) Two particles of masses m_1 and m_2 are located on a frictionless double inclined plane and connected by an inextensible massless string passing over a smooth peg. Using D'Alembert's principle, describe the motion of the masses.

{(1+2)+1}+3

b) i) Explain the significance of Lande's g-factor. Show that

$$g = 1 + \frac{J(J+1) + S(S+1) - L(L+1)}{2J(J+1)}.$$

ii) The time-dependent wavefunction of a particle of mass m moving in one dimension under the influence of a potential V(x) is given by

$$\begin{aligned}\psi(x,t) &= \alpha x e^{-\beta x} e^{\frac{i\gamma t}{\hbar}} \quad \text{for} \quad x > 0\\ &= 0 \quad \text{for} \quad x < 0,\end{aligned}$$

where, α, β and γ are constants. For x>0, calculate the potential V(x).

(1+3)+3

c) i) State and explain Moseley's law. Derive Moseley's law from Bohr's theory.

ii) In a two valence electron, the quantum numbers of the electrons are n_1=6, l_1=3,

$s_1=\frac{1}{2}$ and $n_2=5$, $l_2=1$, $s_2=\frac{1}{2}$ respectively. Find the values of L and J in L–S coupling and J in j-j coupling. (1+2)+(2+2)

d) i) Find the commutator $\left[\frac{\partial^2}{\partial x^2}, x\right]$, and hence show that the result may be expressed in terms of the momentum operator as $\left[\hat{p}^2, \hat{x}\right] = -2i\hbar\hat{p}$.

ii) Establish Heisenberg's uncertainty relation between position and momentum in case of a quantum one-dimensional harmonic oscillator, starting from the expression of discrete energy eigenvalues. 3+4

e) Write down the Lagrangian and find the equation of motion of a particle of mass m moving with a velocity v near the surface of earth's rotating co-ordinate system. Briefly explain each term. Also find the expression for the Hamiltonian of the system. 3+2+2

GROUP-D

4. Answer any **four** questions: 10×4=40

a) i) What is anomalous Zeeman effect? Calculate Lande g-factors for different levels and indicate the possible transitions between magnetically split Sodium Dl and D2 lines.

ii) Determine the probability current density for the wave function given by $\psi = A\exp\left(-\frac{\alpha^2 x^2}{2}\right)\exp(ikx)$ evaluating the normalization constant.

(1+4) + (1+4)

b) i) Define generalized co-ordinates and obtain the expression for generalized force and generalized potential.

ii) Derive Newton's second law of motion from Hamilton's principle.

iii) A particle of mass m moves on a plane in the field of force given by $\vec{F} = -\hat{r}\,kr\cos\theta$, where k is constant and $\hat{r}$ is the radial unit vector. Will the angular momentum of the particle about the origin be conserved? Justify your statement. Obtain the differential equation of the orbit of the particle.

3+2+(3+2)

c) i) Define point transformation and canonical transformation. Write down the advantages of canonical transformation.

ii) Show that the transformation $P = q\cot p$, $Q = \log\left(\frac{\sin p}{q}\right)$ is canonical.

iii) Obtain Lagrangian, Hamiltonian and Canonical equations of motion of a charged particle in an electromagnetic field. (1+1+1)+2+(1+1+3)

d) i) State Ehrenfest's theorem. Establish it in case of dynamical variables relating position and momentum of a quantum mechanical particle.

ii) A potential step is defined by V=0 for the region x<0 and $V = V_0$ for x>0. Find the 'reflectance' and 'transmittance' at the potential discontinuity. Show that the incident wave in the negative x-region is totally reflected if the particle has energy 'E' $< V_0$. (1+3)+(2+2+2)

e) i) Show that Maxwell-Boltzmann energy distribution law is a limiting case of Bose-Einstein and Fermi-Dirac statistics.

ii) What is meant by electron gas? Derive the expression for energy distribution of free electrons in metals.

iii) Copper contains 8.5×10^{34} free electrons per cc. Assuming that copper atoms donate one free electron, calculate the Fermi energy. Also find the number of quantum states in the energy range $\epsilon_F \pm kT$ for the free electrons at 300 K in 1cc. of gas. 3+4+3

f) The bobs of two pendulums having equal length and mass are attached by a spring and set in motion. Find the Euler-Lagrange equations of motion of the system. Under the assumption of small oscillation, find the secular determinant and the normal modes of vibration. Also find the general solutions, using orthonormal condition. 3+2+2+3

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