2022
MATHEMATICS
[HONOURS]
Paper : VIII

Full Marks : 50 Time : 2 Hours

The figures in the right-hand margin indicate marks.

Symbols and Notations have their usual meaning.

GROUP–A

(Numerical Analysis)

(Marks : 30)

1. Answer any **two** questions: 1×2=2

   a) What do you mean by degree of precision of a quadrature formula?

   b) Prove that $E = \Delta + 1$.

   c) Find the absolute error, relative error and percentage error in the approximate value x_A=0.333 when the true value $x_T = \frac{1}{3}$.

   d) What is the number of multiplications involved in solving a system of n linear equations with n unknowns by Gauss Elimination method?

2. Answer any **three** questions: 2×3=6

   a) If $f(x) = \frac{1}{x^2}$, whose arguments are a, b, c then prove that $f[a,b,c] = \frac{ab+bc+ca}{a^2b^2c^2}$.

   b) Show that $\Delta \log f(x) = \log\left\{1 + \frac{\Delta f(x)}{f(x)}\right\}$

   c) Define polynomial interpolation. Comment on its uniqueness.

   d) Solve the equation $\frac{dy}{dx} = xy + 1$ when y(0)=1 by Picard's method, upto x^3 as the highest degree term.

   e) Evaluate $\Delta^{10}(1-px)(1-qx^2)(1-rx^3)(1-sx^4)$.

3. Answer any **two** questions: 6×2=12

   a) Explain the basic concepts used in the Gauss elimination approach for solving a linear system of n equations. Show that the number of multiplications and subtractions grows proportional to $\frac{n^3}{3}$ and the number of divisions proportional to $\frac{n^2}{2}$ from the computational effort involved.

b) i) Show that the divided differences of order n are symmetric functions of their arguments.

ii) Show that the n^{th} difference of a real polynomial p(x) of degree n is constant and $(n+1)^{th}$ difference vanishes. 3+3

c) Solve the equation $\frac{dy}{dx} = \frac{x^2}{1+y^2}$ with the initial condition y(0)=0 by Picard's method to obtain y for x=0.15, 0.25 and 0.5 correct to 3 decimal places.

d) Derive Newton-Raphson's method for finding real simple root of $f(x)=0$ and discuss its convergence.

e) Explain the Gauss-Seidel iterative method of solving a system of linear equations. Give sufficient conditions for convergence of the process.

4. Answer any **one** question: $10\times1=10$

a) i) Establish Lagrange's interpolation formula. If $x_0, x_1, x_2,..., x_n$ be the interpolating points and $\omega_i(x)$, i = 0, 1, 2,...n be the Lagrangian functions, then show that $\sum_{i=0}^{n} \omega_i(x) = 1$. 6

ii) Show that $\Delta\binom{n}{x+1} = \binom{n}{x}$, where the forward difference operator Δ operates on n and hence show that

$$\sum_{n=1}^{N}\binom{n}{i} = \binom{N+1}{i+1} - \binom{1}{i+1}.$$ 4

b) i) Construct Lagrange's interpolation polynomial for the function $y = \sin \pi x$, choosing the points

$x_0 = 0,\ x_1 = \frac{1}{6},\ x_2 = \frac{1}{2}$. 4

ii) Show that the maximum error in linear interpolation for f(x) on $[x_1, x_2]$ is given by $\frac{1}{8}(x_1 - x_0)^2 M$, where

$M = \max|f''(\xi)|,\ x_0 \le \xi \le x_1$.

Hence or otherwise, determine an appropriate step-size in the construction of a table of $f(x) = (1+x)^6$ on [0, 1] so that the transaction error remains less than 5×10^{-5}. 3+3

GROUP–B

(Fundamentals of Computer Science and Computer Programming)

(Marks : 20)

5. Answer any **two** questions: 2×2=4

   a) What are keywords in C? Can keywords be used as an identifier?

   b) What do you mean by Machine language and Assembly language?

   c) Write the expression in equivalent C form: $x = y\log_{10}(\cos m) + z\sqrt{\sin^{-1}(|p|)}$.

   d) How a source program is converted into an object program? Explain with a diagram.

6. Answer any **one** question: 6×1=6

   a) i) What is CPU? What do you mean by a software? 2

   ii) Use 2's complement method to compute the difference $(1001.011)_2 - (111.111)_2$. 2

   iii) Illustrate the difference between compiler and interpreter. 2

   b) Write two reasons of using binary numbers, instead of decimal numbers, in the computer memory. Perform subtraction of the following decimal numbers after converting them into binary numbers using binary arithmetic and find the decimal equivalence of the result: $(94.5)_{10} - (43.75)_{10}$. 1+5

7. Answer any **one** question: 10×1=10

   a) i) What is a loop? Why it is necessary in a C program? 2

   ii) How do you choose between 'while' and 'for' loop? 2

   iii) Write a C program to check if the entered year is a leap year or not. 6

   b) i) What is an array? How multidimensional array is different from one-dimensional array? 3

   ii) Write a C program to find the sum of squares of the diagonal elements of a square matrix of order n×n using the concept of array. 7

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