Kandi Raj College – Department of Mathematics – Internal QP – 2021 – $4^{th}$ Semester

HONOURS: **Full marks: 10 + 10 + 10 + 05**

**ANSWER EACH PART IN SEPARATE ANSWER SCRIPTS:**

**CC-8** **[10]**

Answer any TWO questions

1.(i) Show that the Second MV theorem (Bonnet's form) is applicable to $\int_a^b \frac{Sin\,x}{x}dx$ where 0 < a < b < ∞. Also prove that $\left|\int_a^b \frac{Sin\,x}{x}dx\right| \leq \frac{2}{a}$. [2]

(ii) Show that the Second MV theorem (Weirstrass form) is applicable to $\int_a^b \frac{Sin\,x}{x}dx$ where 0 < a < b < ∞. Also prove that $\left|\int_a^b \frac{Sin\,x}{x}dx\right| < \frac{4}{a}$. [3]

2. A sequence of functions defined by $f_n(x) = \frac{nx}{1+n^2x^2}$, $0 \leq x \leq 1$. Show that the sequence $\{f_n\}$ is not uniformly convergent in [0, 1]. [5]

3. Show that the Fourier Series corresponding to $x^2$ on $[-\pi, \pi]$ is [5]

$$\frac{\pi^2}{3} + 4\sum_{n=1}^{\infty}(-1)^n \frac{Cos\,nx}{n^2}.$$

Hence deduce that, $1 + \frac{1}{2^2} + \frac{1}{3^2} + \ldots = \frac{\pi^2}{6}$

**CC-9** **[10]**

Answer any TWO questions

1. Let $f(x,y) = \begin{cases} \frac{x^2y}{x^4+y^2} & ,(x,y) \neq 0; \\ 0, & (x,y) = (0,0). \end{cases}$

Show that $f$ has a directional derivatives at (0,0) in any direction $\beta = (l,m), l^2 + m^2 = 1$ but $f$ is discontinuous at (0,0).

2. Verify Stokes' theorem for a vector field defined by $\vec{F} = (x^2 - y^2)\vec{\imath} + 2xy\vec{\jmath}$ in the rectangular region in the xy plane bounded by the straight lines $x = 0, x = a, y = 0, y = b$.

3. Find $\int\int\int_E \frac{dxdydz}{x^2+y^2+(z-2)^2}$ where $E$ is the region bounded by the sphere $x^2 + y^2 + z^2 = 1$

**CC-10** **[10]**

Answer any TWO questions

1. Let $H$ be the commutator subgroup of a group $G$ then show that $H$ is normal in $G$.
2. If $U$ is an ideal of, let $[R:U] = \{x \in R: rx \in U$ for every $r \in R\}$. Prove that $[R:U]$ is an ideal of $R$ and that contains $U$.
3. Let $(F, +, .)$ be a field and $(\neq 0) \in F$. Define multiplication $\times$ in $F$ by $a \times b = a.u.b$ for $a, b \in$ F. Prove that $(F, +, \times)$ is a field.

**SEC** **[05]**

Answer any ONE question

1. Let $G$ be a graph in which all vertices have degree at least two. Show that $G$ contains a cycle. [5]
2. Show that, for any graph $G$, $\delta(G) \leq d(G) \leq \Delta(G)$, where $\delta(G)$ and $\Delta(G)$ are the minimum and maximum degrees of the vertices of $G$, and $d(G) = \frac{1}{n}\sum_{v \in V} d(v)$ is their *average degree*. [5]

**[FOR STUDENTS OTHER THAN MATHEMATICS HONOURS]**

HONOURS GENERAL (HGE): **Full marks: 10**

Answer any TWO questions **[10]**

1. Solve: $\frac{dy}{dx} = \frac{y+x-2}{y-x-4}$
2. Solve: $(xy^3 + y)dx + 2(x^2y^2 + x + y^4)dy = 0$
3. Reduce the differential equation (xp-y)(x-py)=2p to Clairaut's form by the substitution $x^2 = u$, $y^2 = v$ and find the general and singular solution.