U.G. 4th Semester Examination - 2022

MATHEMATICS

[Other than Mathematics Honours]

Generic Elective Course (GE)

Course Code : MATH-H-GE-T-02

Full Marks : 60 Time : $2\frac{1}{2}$ Hours

The figures in the right-hand margin indicate marks.

The symbols and notations have their usual meanings.

1. Answer any **ten** questions: 2×10=20

a) Find the degree of the differential equation $\left(\frac{d^2y}{dx^2}\right)^{\frac{3}{2}} - 2x\frac{dy}{dx} = sin\left(\frac{d^2y}{dx^2}\right)$.

b) Let $(y-c)^2 = cx$ be the primitive of the differential equation $4x\left(\frac{dy}{dx}\right)^2 + 2x\frac{dy}{dx} - y = 0$. Find the number of integral curves passing through the point (1, 2).

c) If $y_1 = 1 + x$ and $y_2 = e^x$ be two solutions of $\frac{d^2y}{dx^2} + P(x)\frac{dy}{dx} + Q(x)y = 0$ Find $P(x)$.

d) Consider the differential equation $\frac{d^2y}{dx^2} + a\frac{dy}{dx} + by = 0$, where a, b are real

constants. Its characteristic equation has a root m of multiplicity 2. Find the Wronskian of the solutions of the differential equation.

e) Find the solution of the differential equation: $\frac{d^2y}{dx^2} - y = 1, y(0) = 0$ and $y \to$ a finite value as $x \to -\infty$.

f) Find the partial differential equation of the family of all spheres of radius c having centers on the xy -plane.

g) Eliminate the arbitrary function f from $z = e^{ny}f(x-y)$.

h) If $y = x$ is a solution of the differential equation $x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} - y = 0$. Find another linearly independent solution of the given differential equation.

i) Find the singular solution of : $y = px + p^3$.

j) Show that the differential equation of the family of circles of fixed radius r with centers on y axis is $(x^2 - r^2)\left(\frac{dy}{dx}\right)^2 + x^2 = 0$.

k) Find two linearly independent solution of the following differential equation: $4\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + 5y = 0$.

l) Find the number solutions of the initial value problem: $\frac{dy}{dx}-\sqrt{y}=0, y(0)=0$.

m) Find the nature of the partial differential equation:
$h(z_{xx}-z_{yy})-(a-b)z_{xy}=0; a,b,h$ are real constants.

n) Find the characteristics of the partial differential equation: $3z_{xx}+10z_{xy}+3z_{yy}-14z_y=0$.

o) Find the Particular integral of the differential: $(D^3-D)y=e^x+e^{-x}$.

2. Answer any **four** questions: 5×4=20

a) Solve the differential equation:
$(y-2x)dx+(4y+3x)dy=0$. 5

b) Solve the differential equation the by method of variation of parameters:

$(1+x)\frac{d^2y}{dx^2}+x\frac{dy}{dx}-y=(1+x)^2$, given that $y=x, y=e^{-x}$ are independent solutions of its reduced equation. 5

c) Solve the differential equation:

$$2x\frac{d^3y}{dx^3}\cdot\frac{d^2y}{dx^2}=\left(\frac{d^2y}{dx^2}\right)^2-a^2$$ 5

d) Verify that $y=x$ is a solution of the reduced equation of $x^2\frac{d^2y}{dx^2}-x\frac{dy}{dx}+y=x^2$. Solve the equation after reducing it to first order linear equation. 5

e) Solve the differential equation:

$$(1+2x)^2\frac{d^2y}{dx^2}-6(1+2x)\frac{dy}{dx}+16y=8(1+2x)^2$$ 5

f) Reduce the following 1st order partial differential equation to its canonical form and hence find the general solution: $z_x-z_y=z$. 5

g) Solve the differential equation: $e^{p-y}=p^2-1$ 5

3. Answer any **two** question: 10×2=20

a) i) Reduce the following partial different equation to its canonical form:

$$z_{xx}-2\sin(x)\,z_{xy}-cos^2(x)z_{yy}-\cos(x)z_y=0.$$ 5

ii) Solve the differential equation: $(D^2+2)y=x^2e^{3x}+e^x cos2x$. 5

b) i) Find the general and singular solution of differential equation: 5

$(px^2+y^2)(px+y)=(p+1)^2$.

ii) Solve the following system of simultaneous linear differential equation: 5

$$(D+4)x+3y=t$$
$$(D+5)y+2x=e^t$$

c) i) Solve the total differential equation:

$$2(y+z)dx-(x+z)dy+(2y-x+z)dz=0.$$
5

ii) Solve: $\frac{dx}{x(z^2-y^2)}=\frac{dy}{y(x^2-z^2)}=\frac{dz}{z(y^2-x^2)}$. 5

d) i) Solve

$$(y^3x-2x^4)p+(2y^4-x^3y)q=9z(x^3-y^3)$$
by Lagrange's method. 5

ii) Solve $z=\sqrt{p}+\sqrt{q}$ by Charpit's method. 5

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