

Question for Honours Students dated: 13.08.2022

**First Part: Question for Mathematics Honours Students**

Answer all Courses (Papers) in Separate Answer Scripts

**CC-T-03 10**

Answer any Two questions:

1. Prove that the sequence  $\{x_n\}$  defined by  $x_1 = \sqrt{7}$  and  $x_{n+1} = \sqrt{7 + x_n}$ ,  $\forall n \geq 1$  converges to the positive root of the equation  $x^2 - x - 7 = 0$ . 05
2. Show that the unit interval  $[0,1]$  is uncountable. 05
3. Show that the series  $\frac{a+x}{b+x} + \frac{(a+x)(2a+x)}{(b+x)(2b+x)} + \frac{(a+x)(2a+x)(3a+x)}{(b+x)(2b+x)(3b+x)} + \dots$  is convergent if  $a, b, x$  be all positive and  $a < b$ . 05

**CC-T-04 10**

Answer any Two questions:

1. If  $M(x,y) dx + N(x,y) dy = 0$  has one and only one solution, then prove that there exists an infinity of integrating factors. 05
2. Solve  $(2xy^4e^y + 2xy^3 + y)dx + (x^2y^4e^y - x^2y^2 - 3x)dy = 0$  05
3. Find the singular solution of the equation  $x^3 p^2 + x^2 yp + a^3 = 0$  05

Question for Mathematics Honours ends here.....

**Second Part: Question for Other Honours Students**

**GE-T-02 10**

A. Answer any One question: 05

1. If  $u = \log \left( \frac{x^2 + y^2}{\sqrt{x} + \sqrt{y}} \right)$ , then show that,  $x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} = \frac{3}{2}$ . 05
2. If  $y = \sin(m \sin^{-1}x)$ , then show that  $(1 - x^2)y_{n+2} - (2n + 1)xy_{n+1} + (m^2 - n^2)y_n = 0$  05
3. A function  $f: [0, 1] \rightarrow \mathbb{R}$  is defined by 05

$$f(x) = \begin{cases} x, & x \in \mathbb{Q} \cap [0, 1] \\ 1 - x, & x \in \mathbb{R} \setminus \mathbb{Q} \cap [0, 1] \end{cases}$$

Show that 'f' is continuous at 1/2 and discontinuous at every other points in  $[0, 1]$ .

B. Answer any One question: 05

1. Solve:  $(xy^3 + y)dx + 2(x^2y^2 + x + y^4)dy = 0$  05
2. Find the general and singular solution of the equation  $x^3 p^2 + x^2 yp + a^3 = 0$  05
3. Solve the differential equation  $\frac{d^3y}{dx^3} + 3 \frac{d^2y}{dx^2} - 4y = x e^{-2x}$  05

Question for Other Honours ends here.....